# Portfolio Choice Analysis in a Multi-country Macro Model

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**Abstract**

This paper examines portfolio choice in a dynamic stochastic general equilibrium model with trade and financial linkages across 43 countries. I conduct comparative statics analysis with this structural model to quantify potential mechanisms of global financial allocation, including risk hedging, risk diversification, risk sharing, and financial friction. For asset home bias, the model predicts that risk hedging is less essential in a multi-country than in a two-country setting. For bilateral asset positions, the model implies that variations in financial friction and asset covariance are major determinants of countries' observed portfolios. Meanwhile, bilateral financial linkages strongly covary with trade linkages across countries. Counterfactual analysis suggests that this covariance is mainly driven by the high correlation of frictions across the two channels of globalization.

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# 1 Introduction

Global financial allocation is an important topic in open economy macro with profound policy implications. Understanding the allocation requires developing multi-country general equilibrium models to quantify the implications of determinants for portfolio choice. However, there have been few quantitative studies in the international macro literature due to the difficulty of solving a portfolio choice problem, especially in a setting with many economies in incomplete markets. This paper fills this gap and investigates asset allocation in a DSGE model with trade and financial linkages across many countries.

I examine mechanisms of cross-country portfolio choice including those proposed in the asset home bias literature (see Coeurdacier and Rey (2013) for a survey). Many home bias papers assume two symmetric countries in complete markets where analytical solutions are obtainable. Nonetheless, financial frictions in the form of trading costs, capital controls, information asymmetry, and tax differentials are major barriers to asset allocation (see, for example, Lewis (1999) and Portes and Rey (2005)). I allow for the existence of financial frictions when solving the portfolio choice problem. Furthermore, the model covers 43 countries which trade with each other in both asset and goods markets subject to frictions. When deciding on financial allocation, countries not only choose between domestic and foreign assets, but also assign weights to each of the foreign assets in their portfolios. The derived portfolios, influenced by cross-country covariances of macro and financial variables, enable general equilibrium analysis of both asset home bias and bilateral asset positions. I use this structural framework to evaluate potential determinants of global financial allocation, which include risk sharing affected by financial frictions, risk hedging against labor income and real exchange rate risks, risk diversification shaped by the asset covariance structure, and the variation of bilateral financial frictions in international markets.

To solve for portfolios with many countries subject to financial frictions, I follow the solution method developed by Devereux and Sutherland (2011) (similar to Tille and van Wincoop (2010)). They combine a second-order approximation of the Euler equations with a first-order approximation of other model equations to determine a steady-state (zero-order) portfolio. It is computationally challenging to perform higher-order approximations of a large-scale DSGE model, especially in a multi-country analysis with numerous variables and equations. To mitigate the challenge, I embed a Ricardian trade model developed by Eaton and Kortum (2002) in a dynamic open economy macro framework. The trade model characterizes intra-temporal allocation across many economies analytically, which allows me to solve for inter-temporal allocation with the linearization method for DSGE models efficiently. This novel approach that combines macro and trade methods greatly simplifies the portfolio choice analysis in a multi-country model, and has the potential for wide applications to other topics in open economy macro.

The model structure incorporates portfolio choice analysis in a multi-country real business cycle model. Countries produce and trade intermediate goods with each other subject to bilateral trade cost. Moreover, they issue equities as claims to capital income net of investment expenditure. There is a representative household in each country which constructs a portfolio of different countries' equities to maximize expected lifetime utility. To reduce the impact of country-specific productivity shocks on consumption, the household's portfolio choice is shaped by the covariance between its consumption and asset returns. Besides, it considers bilateral financial friction when repatriating returns from another country. After providing qualitative analysis of mechanisms for portfolio choice, I calibrate the model to a world economy with 43 countries and the rest of the world for quantitative exercises. In particular, I conduct comparative statics analysis where I turn off mechanisms sequentially. The departure of counterfactual from observed portfolios quantifies the relevance of these mechanisms for countries' asset positions. Moreover, I need only a few sufficient statistics including bilateral trade shares and portfolio weights to examine comparative statics. These statistics already embed the impacts of bilateral trade and financial costs so that we do not need to calibrate these frictions.

I start portfolio analysis with risk hedging mechanism and find it to be less essential for asset home bias in a multi-country than in a two-country setting. In the home bias literature, economists contend that households may favor domestic assets which provide hedging benefits against the fluctuations in domestic labor income and in real exchange rate (RER).[1] I conduct numerical exercises with both 43 countries and 2 countries to evaluate the importance of risk hedging for countries' domestic asset holdings. The numerical results suggest that both labor income risk and real exchange rate risk generate home bias on average across sample countries in a two-country case. Nevertheless, the importance of risk hedging is diminished in a multi-country case. The intuition for this result is that, when households construct a portfolio with 43 instead of 2 assets with non-perfectly correlated returns, any country-specific risk can be easily hedged away by this diversified portfolio. The covariance between domestic fundamentals (including labor

[1] For example, see Obstfeld and Rogoff (2000) and Coeurdacier and Gourinchas (2016). However, theoretical predictions from two-country models and empirical findings are both mixed to rationalize home bias by risk hedging in the literature, as summarized by Coeurdacier and Rey (2013).

income and RER) and the relative return of domestic assets becomes significantly weaker. If risk hedging needs arise, instead of only domestic assets, households can adjust all the assets simultaneously which offer similar hedging benefits in this multi-country setting, where the importance of risk hedging for asset home bias declines.

I proceed to examine mechanisms of bilateral asset positions beyond home bias, and find that financial friction and diversification motive are important determinants of cross-country financial allocation. The international macro literature has proposed risk sharing as a driver for portfolio choice.[2] Nonetheless, risk sharing is potentially impaired by barriers to global financial allocation. To measure market incompleteness caused by the existence of these frictions, I solve for the world portfolio absent financial friction and find this portfolio to deviate remarkably from data. Therefore, markets are not complete to ensure efficient allocation of financial resources, which induces households to consider the asset covariance structure for risk diversification. The model suggests that their portfolios are heavily influenced by the heterogeneity in bilateral financial frictions and asset covariances across countries. In counterfactual exercises where I set each holder country's financial frictions and return covariances with others to be the same as their median level across investment destinations, countries would substantially adjust portfolios from their observed composition: the weight of assets from countries with higher frictions and covariances rises significantly in portfolios. The large departure of counterfactual from observed portfolios implies that financial friction and risk diversification motive play a crucial role in drawing the landscape of global financial allocation.

These mechanisms also explain the strong connection between bilateral financial and trade linkages. In the data I document an empirical regularity that country pairs with stronger trade ties also invest more in each other's assets. If we measure bilateral linkages as the share of goods or assets in each other's total trade volume or portfolio, the correlation between bilateral trade and financial linkages across country pairs is as high as 0.83 in the data. To explain this fact, I conduct counterfactual portfolio analysis and find the correlation to drop to 0.17 in complete markets and turn negative when heterogeneity in financial frictions is turned off. Meanwhile, the correlation does not decline sharply when the hedging of RER risk or risk sharing through RER adjustment is shut down. Based on these results, the high correlation of trade costs and financial frictions is the main driver for synchronous bilateral trade and bilateral financial linkages.[3]

[2] For example, Obstfeld and Rogoff (2000) and Heathcote and Perri (2013).

[3] The correlation of the two frictions across the two channels can be understood from the fact that bilateral trade costs and financial frictions are shaped by common factors such as geographic distance, cultural similarity, information accessibility, and regional economic integration.

This paper contributes to the international macro literature on portfolio choice and home bias, by quantifying the mechanisms of financial allocation across 43 countries with frictional trade and financial linkages. As surveyed by Coeurdacier and Rey (2013), economists have examined risk-hedging motives to explain the skewness of portfolios towards domestic assets (for example, Obstfeld and Rogoff (2000) and Coeurdacier and Gourinchas (2016)). Others have proposed financial frictions such as transaction barriers and information frictions as determinants for investors' portfolio choice (for example, Portes and Rey (2005) and Okawa and van Wincoop (2012)). I incorporate both mechanisms in a general equilibrium model and quantify their impacts on global financial allocation through counterfactual exercises. This paper is particularly related to the works that focus on the implications of trade pattern for financial allocation, including Coeurdacier (2009), Heathcote and Perri (2013), Steinberg (2018), Hu (2020), and Chau (2022). Many of these studies solve for portfolios in complete markets, while this paper generalizes the analysis to a setting with financial frictions. Quantitative predictions from the model suggest that existence and variation of bilateral financial frictions are important drivers for countries' asset positions.

I solve for portfolios with the method developed by Devereux and Sutherland (2011) and Tille and van Wincoop (2010), who use a second-order approximation of the Euler equations to overcome the certainty equivalence of different assets in a first-order approximation. Compared to alternative portfolio techniques, this macro approach does not require separate utility assumptions for agents' intratemporal financial allocation, which is instead determined by endogenous second-moment variables in general equilibrium.[4] Moreover, this portfolio technique offers wide macro applications with its compatibility with DSGE frameworks, which facilitates the understanding of macro implications of global financial allocation. Many works in open economy macro build upon models with a small open economy or two symmetric countries to deliver important and elegant mechanisms. This paper provides an alternative approach by simplifying the quantitative analysis with many countries of uneven sizes linked through multilateral financial and trade relations subject to bilateral frictions. This multi-country approach is valuable for macro economists to examine the patterns and determinants of globalization.

[4]Alternative portfolio techniques include the asset demand system employed by Liu et al. (2022) and the rational inattention logit demand system adopted by Pellegrino et al. (2021), both of whom provide tractable and elegant analytical expressions for intra-temporal financial allocation. The macro approach followed by this paper focuses more on agents' inter- and intra-temporal investment decisions shaped by endogenous second-moment variables under exogenous shocks, which makes it possible to examine mechanisms like risk sharing, risk hedging, and risk diversification in general equilibrium.

# 2 Model

This section consists of two parts. Section 2.1 develops a multi-country model with frictional trade and financial channels. Section 2.2 describes the computation and calibration strategies for the model.

## 2.1 Setup

The world comprises countries of uneven sizes indexed by $i = 1, 2, ..., I$. There is a representative household in each country whose objective is to maximize lifetime utility

$$\max E_0 \sum_{t=0}^{\infty} \nu_t \left( \frac{C_{i,t}^{1-\gamma}}{1-\gamma} - \kappa_1 \frac{L_{i,t}^{1+\kappa_2}}{1+\kappa_2} \right), \tag{1}$$

where $C_{i,t}$ and $L_{i,t}$ are consumption and labor supply respectively, $\gamma$ is the coefficient of relative risk aversion, $\kappa_1$ is a multiplier for labor supply, and $\kappa_2$ is the inverse of the Frisch elasticity. Moreover, I follow Devereux and Sutherland (2009) to introduce an endogenous discount factor $\nu_t$ which satisfies

$$\nu_0 = 1, \qquad \nu_{t+1} = \nu_t \beta(C_{i,t}) \qquad \text{with} \qquad \beta(C_{i,t}) = \omega_i C_{i,t}^{-\psi}, \tag{2}$$

where $0 \leq \psi < \gamma$ and $\omega_i$ is a country-specific multiplier. This endogenous discount factor, whose steady state level is denoted as $\bar{\beta}$, is introduced to ensure a stationary wealth distribution in incomplete markets, otherwise even transitory shocks may have permanent impacts on wealth such that the steady state is indeterminate in a linear approximated model.[5]

Each country produces a final good consisting of a continuum of intermediate goods $u$ traded across countries

$$Q_{i,t} = \int_0^1 [q_{iu,t}(u)^{\frac{\epsilon-1}{\epsilon}} du]^{\frac{\epsilon}{\epsilon-1}}, \tag{3}$$

where $\epsilon$ is the elasticity of substitution in the CES aggregator. The final good can be spent on consumption $C_{i,t}$, capital investment $IV_{i,t}$, or for the production of intermediate goods whose nominal output is $Y_{i,t}$. The optimal consumption-investment decision is

[5] See a detailed discussion by Schmitt-Grohé and Uribe (2003). An alternative assumption to endogenous discount factors to obtain a stationary wealth distribution is the life-cycle elements introduced by Tille and van Wincoop (2010) who assume that agents liquidate wealth with a probability every period.

characterized by the Euler equation

$$C_{i,t}^{-\gamma} = \beta(C_{i,t})E_t[\frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}}((1-\delta)P_{i,t+1} + \frac{\partial Y_{i,t+1}}{\partial K_{i,t+1}})], \tag{4}$$

where $P_{i,t}$ is the price of the final good, $\delta$ is the depreciation rate of capital $K_{i,t}$. The law of motion for capital will be given by

$$K_{i,t+1} = (1-\delta)K_{i,t} + IV_{i,t}. \tag{5}$$

Country $i$'s technology of producing $u$, denoted as $Z_{i,t}(u)$, is drawn from a Fréchet distribution as in the Eaton and Kortum (2002) model[6]

$$\Pr(Z_{i,t} \leq z) = \exp(-T_{i,t}z^{-\theta}), \tag{6}$$

where $T_{i,t}$ captures the central tendency of the distribution which determines the average productivity in country $i$ at $t$ and $\theta$ governs the dispersion of the distribution.

To characterize the risk of the world economy for portfolio analysis, I follow the international business cycle literature to assume that country-level productivity is subject to stochastic shocks.[7] Specifically, $T_{i,t}$ follows an AR(1) process over time

$$T_{i,t} = \rho T_{i,t-1} + (1-\rho)\bar{T}_i + \epsilon_{i,t}, \tag{7}$$

where $\bar{T}_i$ is the productivity level in the steady state of the economy, shocks $\epsilon_{i,t}$ are drawn from a joint normal distribution with a zero mean and cross-country covariance $\Sigma_T$.

Production of intermediate goods in country $i$ combines labor $L_i$, capital $K_i$, and $i$'s final good. The prices of these inputs are wage $w_i$, capital rental fee $r_i$, and price of the final good $P_i$ respectively. Let $\tau_{ij}$ be the iceberg trade cost for exports to $j$, and the price

[6]There are three major benefits of using the Eaton and Kortum (2002) (EK) framework. First, the model characterizes the world trade pattern with more theoretical underpinnings — including Ricardian productivity, trade cost, market size, factor intensity, and input-output linkages — than standard macro models. Second, countries' productivity consistent with their trade patterns and relevant for their risk characteristics can be calibrated from bilateral trade data based on the EK model. Third, intra-temporal allocation across many countries is determined by the gravity trade system in a closed form, which makes this large-scale DSGE framework significantly easier to solve. See more details in the computation section.

[7]For example, canonical frameworks by Mendoza (1991) and Backus et al. (1992) assume an AR(1) process for country-level productivity. Besides productivity shocks, this model can be adapted to accommodate other risks which drive countries' output fluctuations that induce households to construct portfolios for risk sharing. Future extensions can also examine shocks originated from the financial sector, which will further enrich the mechanisms of comvement between trade and financial channels.

of a specific intermediate good $u$ exported from $i$ to $j$ at $t$ will be

$$p_{ij,t}(u) = \frac{\tau_{ij}(r_{i,t}^{\mu} w_{i,t}^{1-\mu})^{\eta} P_{i,t}^{1-\eta}}{Z_{i,t}(u)}, \tag{8}$$

where $1-\mu$ is labor share and $1-\eta$ is the share of the final good in production. In this Eaton and Kortum (2002) model, the share of $i$'s goods in $j$'s expenditure under the Fréchet distribution is

$$\pi_{ij,t} = \frac{T_{i,t}[\tau_{ij}(r_{i,t}^{\mu} w_{i,t}^{1-\mu})^{\eta} P_{i,t}^{1-\eta}]^{-\theta}}{\Phi_{j,t}}, \quad \textit{where} \quad \Phi_{j,t} = \sum_{k=1}^{I} T_{k,t}[\tau_{kj}(r_{k,t}^{\mu} w_{k,t}^{1-\mu})^{\eta} P_{k,t}^{1-\eta}]^{-\theta}, \tag{9}$$

while $\Phi_{j,t}$ is linked to price in country $j$ through a Gamma function $\Gamma = \Gamma(\frac{1-\epsilon}{\theta}+1)^{\frac{1}{1-\epsilon}}$:

$$P_{j,t} = \Gamma \Phi_{j,t}^{-\frac{1}{\theta}}. \tag{10}$$

Let $X_{j,t}$ denote country $j$'s expenditure

$$X_{j,t} = (1-\eta)Y_{j,t} + P_{j,t}(C_{j,t} + IV_{j,t}) \tag{11}$$

and $Y_{i,t}$ be country $i$'s nominal output, then $i$'s goods market clearing condition is

$$Y_{i,t} = \sum_{j=1}^{I} \pi_{ij,t} X_{j,t}. \tag{12}$$

Households have access to international financial markets where assets from different countries are traded. I follow the asset home bias literature including Coeurdacier and Rey (2013) and Heathcote and Perri (2013) to assume that countries issue equities whose dividends are claims to capital income net of investment expenditure[8]

$$d_{i,t} = \eta\mu Y_{i,t} - P_{i,t} IV_{i,t}, \tag{13}$$

[8]This paper focuses on equities for which I have data for the calibration of portfolio weights. If there are bond data which can be merged with the equity data with consistent measure and coverage, it will be very interesting to introduce both assets in the model and compare their risk sharing and hedging benefits. A great example will be Coeurdacier and Gourinchas (2016) who find bonds to be the main hedging instrument for the real exchange rate risk, which explains equity home bias driven by different assets' conditional hedging positions.

which together with equity prices $q_{i,t}$ define financial returns

$$R_{i,t+1} = \frac{d_{i,t+1} + q_{i,t+1}}{q_{i,t}}. \tag{14}$$

Markets are incomplete due to barriers to global financial investment. In particular, financial frictions potentially vary across country pairs, which justifies the gravity model of capital flows documented by Portes and Rey (2005). I follow Heathcote and Perri (2004) and Aviat and Coeurdacier (2007) by introducing financial friction as an iceberg transaction cost $f_{ij}$, such that the household in country $i$ expects to collect $e^{-f_{ij}}R_{j,t+1}$ when repatriating asset returns from country $j$. Besides, these frictions are second-order in magnitude (proportional to the variance of shocks) to be consistent with the solution method for portfolio choice in a DSGE framework developed by Devereux and Sutherland (2011) and Tille and van Wincoop (2010) (DSTW).[9] Acknowledging that assets are distinguishable by their risk characteristics, these authors combine a second-order approximation of the Euler equation

$$\frac{C_{i,t}^{-\gamma}}{P_{i,t}} = \beta(C_{i,t})E_t[\frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}}R_{i,t+1}] = \beta(C_{i,t})E_t[\frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}}e^{-f_{ij}}R_{j,t+1}], \qquad \forall i, j \in \{1, ..., I\}. \tag{15}$$

with a first-order approximation of other equations in the model in order to determine a zero-order (i.e. steady-state) portfolio.

Let $\lambda_{ik,t}$ be $i$'s purchase of $k$'s assets at the end of period $t$, $i$'s budget constraint is

$$P_{i,t}C_{i,t} + \sum_{k=1}^{I} \lambda_{ik,t}q_{k,t} = w_{i,t}L_{i,t} + \sum_{k=1}^{I} \lambda_{ik,t-1}(q_{k,t} + d_{k,t}). \tag{16}$$

[9] Financial frictions are modeled as transaction costs on foreign returns following the tradition of the international macro literature and will be quantified as frictions that cause market incompleteness in the quantitative analyses of the model. I do not take a strong stand on the underlying structure of these barriers to financial investment, which may take many forms in the real world. For example, $f_{ij}$ in this model can reflect a mix of worldwide factors including global financial liquidity, country-specific factors including capital account openness, and pair-specific factors including geographic distance and bilateral financial agreements. It can take alternative forms such as informational frictions, as Okawa and van Wincoop (2012) find that these types of frictions yield comparable implications for the gravity model of financial flows. The frictions are second-order in magnitude to retain the certainty equivalence of assets to the first order approximation of the model, so that the solution method for portfolio choice developed by DSTW can be applied. Quantitative exercises in the next section show that these frictions generate sizable impacts on portfolios.

Meanwhile, the supply of assets issued by any country $k$ is normalized at unity

$$\sum_{i=1}^{I} \lambda_{ik,t} = 1. \tag{17}$$

If we introduce net holdings defined as

$$\alpha_{ii,t} = q_{i,t}(\lambda_{ii,t} - 1), \qquad \alpha_{ij,t} = q_{j,t}\lambda_{ij,t}, \qquad \forall j \neq i, \tag{18}$$

we can re-write the budget constraint as

$$P_{i,t}C_{i,t} + \sum_{k=1}^{I} \alpha_{ik,t} = \underbrace{\eta(1-\mu)Y_{i,t}}_{\text{Labor Income}} + \underbrace{\eta\mu Y_{i,t} - P_{i,t}IV_{i,t}}_{\text{Domestic Dividend Income}} + \sum_{k=1}^{I} \alpha_{ik,t-1}e^{-f_{ik}} \underbrace{\frac{q_{k,t} + d_{k,t}}{q_{k,t-1}}}_{R_{k,t}} \tag{19}$$

and the asset market clearing condition as

$$\sum_{i=1}^{I} \alpha_{ik,t} = 0. \tag{20}$$

It follows from the budget constraint that country $i$'s evolution of wealth is

$$D_{i,t} = D_{i,t-1}e^{-f_{iI}}R_{I,t} + \sum_{k=1}^{I-1} \alpha_{ik,t-1}(e^{-f_{ik}}R_{k,t} - e^{-f_{iI}}R_{I,t}) + \eta Y_{i,t} - P_{i,t}(C_{i,t} + IV_{i,t}), \tag{21}$$

where the net wealth position $D_{i,t}$ is the sum of bilateral holdings

$$D_{i,t} = \sum_{k=1}^{I} \alpha_{ik,t}. \tag{22}$$

To solve the portfolio choice problem in this DSGE framework, I follow the technique developed by Devereux and Sutherland (2011). Households' equilibrium portfolios derived from the method are influenced by second-moment variables, which reflect cross-country covariances of macro and financial variables in general equilibrium. Appendix C.1 characterizes the portfolio choice problem in this multi-country model. Appendix C.2 shows the second-moment variables that influence households' portfolio choice. Section 3 will examine the mechanisms of cross-country financial allocation influenced by these second-moment variables in great detail.

To conclude the description of the model setup, the general equilibrium of the model consists of a set of prices and quantities such that 1) households choose consumption and investment to maximize expected lifetime utility, 2) firms set output and price to maximize profit, and 3) factor, goods, and asset markets all clear.[10] The steady state of the economy is obtained by dropping stochastic shocks and time subscripts in all the equations, which characterizes the stationary equilibrium of this DSGE model.

## 2.2 Computation and Calibration

This section describes two strategies employed in this paper to facilitate portfolio choice analysis in a large-scale DSGE model. The first is system reduction to make the computation more efficient, and the second is comparative statics with sufficient statistics.

Solving for equilibrium portfolios in this DSGE model requires loglinearizing the model around a steady state to predict the dynamic responses of endogenous variables to stochastic shocks (see Appendix D.1 for details). This is computationally challenging to implement in a large-scale multi-country framework.[11] To mitigate this challenge, I reduce the linear system to include state variables and forward-looking control variables only when performing eigen-decomposition of the dynamic model. This idea of system reduction, introduced by the macro literature including King and Watson (2002) and Hernandez (2013), yields substantial gains in efficiency. I adapt their idea to the international context by embedding a gravity trade model in an open economy macro framework. The gravity trade model efficiently characterizes intra-temporal allocation across many economies, which makes the system reduction method easier to implement. Appendix D.2 provides more details about the implementation of this method.

The second challenge lies in the difficulty of calibrating bilateral trade and financial frictions across all the country pairs, which would often require additional assumptions and take much effort. To alleviate this problem, I follow the trade literature to study comparative statics with sufficient statistics. In particular, I start with countries' observed bilateral trade and portfolio shares, which are sufficient statistics that already embed existing bilateral trade and financial frictions, and conduct comparative statics analysis

[10]The goods and asset market clearing conditions are given by Equations 12 and 20 respectively. The factor market clearing condition ensures that households' labor supply and capital accumulation decided by utility maximization equal firms' demand for labor and capital driven by profit maximization.

[11]Computation is challenging because the coefficient matrices that cover the world economy are very large with numerous equations and variables. Besides, the matrices are badly scaled with countries' uneven sizes and sparse bilateral linkages. Inverting such a large matrix to compute the eigenvalues of the linear system requires high computing power and may sometimes give inaccurate numerical results.

by shutting down potential determinants for portfolio choice sequentially. The departure of counterfactual from observed portfolios allows me to quantify different mechanisms of global financial allocation without having to calibrate existing trade or financial frictions across all the countries.

The model is calibrated to a world economy with 43 countries (listed in Table B.1) plus the rest of the world (ROW). The time-averaged values of the following variables over the sample period 2001-2021 are used to calibrate the steady state of the economy. On the real side, I obtain data for countries' GDP from the Penn World Table (PWT) and bilateral trade shares across countries from the Direction of Trade Statistics. On the financial side, I obtain bilateral portfolio weights from Factset/Lionshare, a dataset that covers cross-country equity holdings of institutional investors (see Appendix B for details). $\check{D}_i = \frac{\bar{D}_i}{\bar{Y}_i}$ as countries' equilibrium wealth is calibrated to the data from the World Bank, which reports trade balance that corresponds to

$$TB_{i,t} = Y_{i,t} - X_{i,t} = \eta Y_{i,t} - P_{i,t}(C_{i,t} + IV_{i,t}). \tag{23}$$

This trade balance as shares of GDP is relatively stable over time in the data whose time-averaged value is denoted as $tb_i = \frac{\bar{TB}_i}{\bar{Y}_i}$. We can plug it in countries' wealth constraint (21) in the steady state to recover $\check{D}_i$:[12]

$$\check{D}_i = \frac{tb_i}{1 - \frac{1}{\bar{\beta}}}. \tag{24}$$

The risks of the world economy are driven by productivity fluctuations. Therefore, I estimate countries' dynamic productivity consistent with the Eaton and Kortum (2002) model following Levchenko and Zhang (2014) and compute its persistence ($\rho = 0.85$), mean, and covariance matrix (see Appendix B.2). Other parametric assumptions are standard in the literature: coefficient of relative risk aversion $\gamma = 2$, annual discount factor $\bar{\beta} = .9$, inverse of the Frisch elasticity of labor supply $\kappa_2 = 2$, capital depreciation rate $\delta = .1$, trade elasticity $\theta = 4$ based on Simonovska and Waugh (2014)'s estimates,

[12] Devereux and Sutherland (2009) discuss the potential pitfalls of calibrating the steady state wealth to an exogenously determined level which may miss mechanisms such as precautionary saving and risk sharing influenced by second moments in the model, and hence solve for the stochastic (risky) steady state from the second-order approximation of the model instead. But since this model also embeds financial frictions which influence consumption, it will be difficult to disentangle these frictions and endogenous second moments following their approach. Therefore, I calibrate wealth to an empirical moment, and shut down different mechanisms by excluding relevant second moments to evaluate their quantitative importance for observed portfolios.

elasticity of discount factor $\psi = 0.01$ following Devereux and Sutherland (2009), share of intermediate input in production $\eta = .312$ following Dekle et al. (2007), and share of labor input $1-\mu$ as country-specific labor income share from the PWT.

Several quantitative exercises will compare model predictions in settings with 2 and with 43 countries. For the former, I collapse the multi-country to a two-country model where each country is treated as the domestic economy and the sum of all the other countries from this country's perspective as the foreign economy. Following this rule, I calculate the two-by-two matrices of financial and trade shares, and re-estimate the productivity of domestic and foreign economies based on country sizes and trade flows.

The appendix provides more computational details: B introduces data sources and calibration strategies. D.1 applies DSTW's solution method to a multi-country portfolio analysis. D.2 combines system reduction with Uhlig (1995)'s toolkit to solve the model.

# 3 Portfolio Choice Analysis

This section provides qualitative and quantitative analyses with the model to elucidate mechanisms of portfolio choice in a multi-country framework. I will examine both domestic asset holdings and bilateral asset positions across countries.

## 3.1 Qualitative Analysis

To highlight the implications of multinational linkages for equilibrium portfolios, I start with the predictions from the international macro literature on the topic of asset home bias. Many home bias papers assume two symmetric countries in a world with perfect risk sharing.[13] Under the assumption, Coeurdacier and Rey (2013) derive a general expression for the share of domestic asset in a country's portfolio

$$\grave{\alpha}_{ii} = \underbrace{\frac{1}{2}}_{\text{Risk sharing (diversification)}} \underbrace{- \frac{1}{2}\frac{1-\mu}{\mu}\frac{\text{cov}(\widetilde{wL}_{i/j}, \widetilde{R}_{i/j})}{\text{var}(\widetilde{R}_{i/j})}}_{\text{Hedging labor income risk}} + \underbrace{\frac{1}{2}\frac{1-1/\gamma}{\mu}\frac{\text{cov}(\widetilde{P}_{i/j}, \widetilde{R}_{i/j})}{\text{var}(\widetilde{R}_{i/j})}}_{\text{Hedging RER risk}}, \quad (25)$$

[13] Risk sharing is achievable under various assumptions. For example, Coeurdacier and Rey (2013) follow the Backus and Smith (1993) condition implied by complete markets when deriving portfolios. Coeurdacier and Gourinchas (2016) show markets are locally complete to support risk sharing if the set of asset returns spans the space of shocks.

where cov and var represent covariance and variance respectively, and $\widetilde{A}_{i/j}$ denotes the log-deviation of any variable $A$'s cross-country ratio from its steady-state value

$$\widetilde{A}_{i,t} = \ln(\frac{A_{i,t} - \bar{A}_i}{\bar{A}_i}), \qquad \widetilde{A}_{i/j,t} = \widetilde{A}_{i,t} - \widetilde{A}_{j,t} \quad \text{for} \quad A_{i/j,t} = A_{i,t}/A_{j,t} \quad \forall i, j \in \{1, ..., I\}. \tag{26}$$

In the expression of domestic asset holding (25), the first term reflects households' incentive to diversify portfolios across different countries' assets to reduce the impact of country-specific risks on consumption (as in Lucas (1982)). The second term captures the hedging of labor income risk, which induces households to hold assets whose return decreases with domestic labor income to avoid the simultaneous shortfall in financial and labor income (Baxter and Jermann (1997) and Heathcote and Perri (2013)). The third term represents the hedging of real exchange rate (RER) risk which motivates risk-averse households to hold assets whose return increases with the domestic price level to sustain purchasing power (Kollmann (2006) and Coeurdacier (2009) among others).

This model departs from these home bias papers along two dimensions. First, I acknowledge the existence of financial frictions when solving for portfolios. International risk sharing is hence not perfect and the first term in Equation 25 is no longer a constant that reflects a country's relative size in the world economy. Instead, households need to consider second-moment variables in the financial market including asset covariances and market frictions when constructing the optimal portfolio for risk diversification. Second, in this framework with $I$ economies, there are $I$-1 foreign countries instead of 1. The correlations between the relative return of individual assets and macro fundamentals are lower in this setting while the diversification benefits offered by a large group of assets with non-perfectly correlated returns are higher. Therefore, risk sharing (diversification) becomes relatively more important than risk hedging in determining portfolio choice in this multi-country framework with incomplete markets than in a two-country framework with complete markets.

To examine the mechanisms of financial allocation in this multi-country framework, I evaluate a general portfolio equation (see Appendix C.2 for its derivation):

$$\frac{\gamma}{\zeta} E_t[\eta \widetilde{Y}\widetilde{R}' + (\frac{\zeta}{\gamma} - \zeta - s)\widetilde{P}\widetilde{R}' - s\widetilde{I}\widetilde{R}' - \widetilde{D}\widetilde{R}' + \widetilde{Z}\widetilde{R}' + \check{\alpha}\widetilde{R}\widetilde{R}'] = \frac{1}{2}F. \tag{27}$$

From this equation, cross-country covariance matrices relevant for portfolio choice are the covariances between excess asset returns and countries' relative macro variables, including with income ($\widetilde{Y}\widetilde{R}'$), price ($\widetilde{P}\widetilde{R}'$), investment ($\widetilde{I}\widetilde{R}'$), wealth ($\widetilde{D}\widetilde{R}'$), and the covariance-variance structure of asset returns ($\widetilde{R}\widetilde{R}'$). $\widetilde{Z}$ represents the numeraire country's excess consumption, influenced by $\zeta$ and $s$ which are countries' equilibrium consumption-to-output and investment-to-output ratios respectively. The portfolio matrix $\breve{\alpha}$ will also be influenced by the matrix of financial frictions $F$.

I examine comparative statics of how portfolio choice $\breve{\alpha}$ is influenced by these second-moment variables. The coefficients of the variables in Equation 27 imply their signs of comovement are

$$\text{(i)}\frac{\partial\breve{\alpha}}{\partial\widetilde{Y}\widetilde{R}'} < 0, \quad \text{(ii)}\frac{\partial\breve{\alpha}}{\partial\widetilde{P}\widetilde{R}'} > 0, \quad \text{(iii)}\frac{\partial\breve{\alpha}}{\partial\widetilde{I}\widetilde{R}'} > 0, \quad \text{(iv)}\frac{\partial\breve{\alpha}}{\partial\widetilde{D}\widetilde{R}'} > 0, \quad \text{(v)}\frac{\partial\breve{\alpha}}{\partial\widetilde{R}\widetilde{R}'} < 0. \tag{28}$$

To provide intuition for (i) which predicts that households would favor assets whose returns decrease with their country's domestic income, I decompose aggregate income $Y$ into labor income $wL$ and capital income $rK$ which decides domestic return $R$.[14] The former is relevant for risk hedging, since households would prefer assets whose returns decrease with domestic labor income to hedge against the labor income risk. The latter is relevant for risk diversification, since households would choose assets whose returns are less correlated with theirs to diversify risks. Hence, both risk hedging and risk diversification motives account for prediction (i). Meanwhile, (ii) can be explained by the RER risk hedging of risk-averse households, who would prefer assets with returns increasing with the domestic price level. Such assets provide hedging benefits for households to sustain purchasing power when local goods are relatively more expensive. (iii) states that households should choose assets whose returns increase with domestic investment expenditure. As is argued by Heathcote and Perri (2013), this cross-country investment financing facilitates risk sharing. Moreover, capital accumulation, by changing the comovement between labor and financial income, also shifts the optimal hedging position against labor income risk. (iv) predicts that households would prefer assets whose returns increase with relative wealth fluctuations. Since wealth fluctuates over time due to time-variation in portfolio returns, this term reflects a hedge against changes in future expected portfolio returns as discussed by Tille and van Wincoop (2010). (v) considers the asset covariance-variance structure and predicts that households assign more portfolio weights to the assets whose returns are less correlated with others' for risk diversification. This term is different from (i) which only considers the covariance with domestic asset return for diversification. $\widetilde{R}\widetilde{R}'$ covers the covariance across all the assets, hence it captures 'multilateral resistance'

[14] Since dividends are claims to capital income net of investment as defined in Equation 13. Dividends together with capital gains define asset returns (Equation 14).

beyond bilateral correlation in the gravity model of international financial flows, consistent with the analysis by Okawa and van Wincoop (2012) and Bergin and Pyun (2016). This covariance structure will determine the portfolio adjustment across assets if any risk hedging or sharing needs arise.

After qualitative analysis, I conduct quantitative exercises to evaluate the impacts of these potential determinants on portfolio choice.

## 3.2 Asset Home Bias

I start quantitative analyses with the risk hedging mechanism from the asset home bias literature summarized by Coeurdacier and Rey (2013). In particular, I compare numerical results in 2- and 43-country frameworks to highlight the differences of predictions.

The proposed risk-hedging mechanism in 25 is affected by the covariances of variables which appear as second moments in 27. To compute these second moments, I solve for the first-order dynamics of endogenous variables in response to exogenous shocks with the linearization method for DSGE frameworks and calculate the product of variables, which will appear as second moments in portfolio equations.[15] Comparative statics analysis, with second moments relevant for different mechanisms excluded from portfolio equations, allows me to examine the importance of these mechanisms for countries' observed asset positions. I start with a general portfolio equation:

$$\frac{\gamma}{\zeta}E_t[\eta\widetilde{Y}\widetilde{R}' + (\frac{\zeta}{\gamma} - \zeta - s)\widetilde{P}\widetilde{R}' - s\widetilde{I}\widetilde{R}' - \widetilde{D}\widetilde{R}' + \widetilde{Z}\widetilde{R}' + \breve{\alpha}\widetilde{R}\widetilde{R}'] = \frac{1}{2}F \tag{29}$$

which is calibrated to the data including observed portfolios, and turn off potential mechanisms to derive counterfactual portfolios.[16]

First, to turn off the hedging against labor income risk, I exclude labor income from households' wealth constraint and rewrite the portfolio equation as

$$\frac{\gamma}{\zeta}E_t[\mu\eta\widetilde{Y}\widetilde{R}' + (\frac{\zeta}{\gamma} - \zeta - s)\widetilde{P}\widetilde{R}' - s\widetilde{I}\widetilde{R}' - \widetilde{D}\widetilde{R}' + \widetilde{Z}\widetilde{R}' + \breve{\alpha}\widetilde{R}\widetilde{R}'] = \frac{1}{2}F, \tag{30}$$

where the coefficient of $\widetilde{Y}\widetilde{R}'$ changes from $\eta$ in Equation 29 to $\mu\eta$ in Equation 30, whose

[15] See Appendix D.1 for details. In particular, Equations D.5 through D.11 describe Devereux and Sutherland (2011)'s method to compute the product of consumption differential and asset returns in portfolio equations. This method applies to the calculation of other second moments for portfolio analysis.

[16] As mentioned in Section 2.2, the calibrated moments include countries' bilateral trade and portfolio shares which already incorporate the impacts of trade and financial frictions. These frictions covary with second moments in the calibrated model.

difference $(1-\mu)\eta$ reflects labor share in output $(\frac{\bar{w}\bar{L}}{\bar{Y}})$. Subtracting this labor income leaves portfolios unaffected by the covariance between labor and financial income. The difference between Equations 29 and 30 suggests that, the counterfactual portfolio $\breve{\alpha}^{ctf1}$ absent labor income risk deviates from the original portfolio observed in the data $\breve{\alpha}^{org}$ by

$$\frac{\gamma}{\zeta}E_t[(1-\mu)\eta\widetilde{Y}\widetilde{R}' + (\breve{\alpha}^{org} - \breve{\alpha}^{ctf1})\widetilde{R}\widetilde{R}'] = 0, \tag{31}$$

where on the right of the equation is a zero because financial frictions are common in original and counterfactual scenarios to be cancelled out, so that we do not need to calibrate these frictions for this numerical exercise. Second moments $\widetilde{Y}\widetilde{R}'$ and $\widetilde{R}\widetilde{R}'$ are solved with the linearization method for this DSGE model as discussed earlier.

Second, to shut down the hedging of RER risk, I assume $\gamma = 1$ for the coefficient of $\widetilde{P}\widetilde{R}'$ under which assumption the portfolio equation turns into[17]

$$\frac{\gamma}{\zeta}E_t[\eta\widetilde{Y}\widetilde{R}' - s\widetilde{P}\widetilde{R}' - s\widetilde{I}\widetilde{R}' - \widetilde{D}\widetilde{R}' + \widetilde{Z}\widetilde{R}' + \breve{\alpha}\widetilde{R}\widetilde{R}'] = \frac{1}{2}F, \tag{32}$$

where the coefficient of $\widetilde{P}\widetilde{R}'$ in Equation 29 is $\frac{\zeta}{\gamma} - \zeta - s = -s$ under log-utility $(\gamma = 1)$. $\gamma$ as the coefficient of risk aversion decides whether households would like to increase income to sustain purchasing power or to reduce consumption when local goods are more expensive. For households with log-utility, the two forces offset each other and leave a portfolio, which generates financial income, independent of fluctuations in RER. To solve for the portfolio in this case $(\breve{\alpha}^{ctf2})$, I take the difference between 29 and 32 which implies

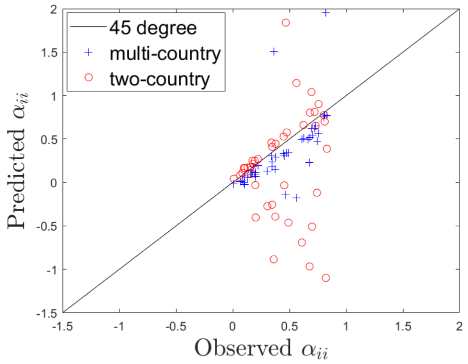
$$\frac{\gamma}{\zeta}E_t[(\frac{1}{\gamma} - 1)\zeta\widetilde{P}\widetilde{R}' + (\breve{\alpha}^{org} - \breve{\alpha}^{ctf2})\widetilde{R}\widetilde{R}'] = 0. \tag{33}$$

Derived from 31 and 33, the shifts of these counterfactual from original portfolios quantify the contribution of risk hedging mechanism to countries' observed asset positions.

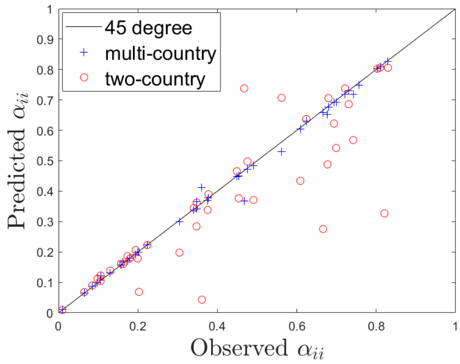
Figure 1 and Table 1 present the numerical results for the impacts of risk hedging on home bias. The weight of domestic assets in portfolios from the Factset/Lionshare data has a median value of 0.378 across sample countries. When portfolio choice abstracts from labor income and RER risk considerations, this median value decreases to 0.171 and 0.234 respectively in a two-country case. Based on these median values, labor income and RER risks account for 55% and 38% observed asset home bias. This finding that the

[17] It is worth noting that we cannot assume log-utility $(\gamma = 1)$ for the whole equation, since that will affect other terms including the hedging against expected portfolio returns (iv in 28).

Figure 1: Domestic Asset Holding and Risk Hedging




(a) No Labor Income Risk Hedging




(b) No RER Risk Hedging

This figure presents the share of domestic assets in countries' portfolios when risk hedging is turned off. Domestic shares observed in the data are on the horizontal axis and model-predicted shares absent labor income or RER risk hedging are on the vertical axis. Red circles and blue stars represent predictions from settings with 2 and 43 countries respectively. See calibration strategies of both settings in Section 2.2, cross-country median values and standard deviations summarized in Table 1, and complete results by country in Table A.1.

two risks would tilt portfolios towards domestic assets on average is consistent with the predictions from many existing papers in the home bias literature. However, the median value from this analysis is not informative about variations in the direction or magnitude of predictions across sample countries. About half of the 43 countries exhibit weaker home bias when either risk hedging is turned off. While the majority of countries do not adjust portfolios by a great magnitude, several countries' predicted portfolios deviate significantly from observed ones, which suggests that risk hedging plays an important role in explaining their home bias in a two-country case.

To diagnose this variation, I examine the underlying mechanism for portfolio choice driven by second moments in the model. When risk hedging is turned off, the shift of an asset position from its original level is determined by the covariance-variance terms in 25:

$$\frac{\text{cov}(\widetilde{wL}_{i/I}, \widetilde{R}_{i/I})}{\text{var}(\widetilde{R}_{i/I})}, \qquad \frac{\text{cov}(\widetilde{P}_{i/I}, \widetilde{R}_{i/I})}{\text{var}(\widetilde{R}_{i/I})}. \tag{34}$$

These two terms represent the 'hedge ratio' against labor income and RER risks respectively. The greater their absolute value is, the larger is the portfolio change when risk hedging is shut down. The sign and magnitude of these hedge ratios vary across countries as reported in Table A.1 and summarized in Table 1, which explains the large cross-country variation in portfolio adjustment. The second moments in these ratios are shaped by many factors not fully incorporated in a symmetric two-country complete-market model, including uneven country sizes, asymmetric trade shares with foreign countries, covariance structure of productivity shocks, and magnitude of financial frictions relative to second moments. Cross-country differences of these factors contribute to the large variation in the importance of risk hedging mechanism for asset home bias. Therefore, the departure of predicted from observed asset home bias is more substantial for some countries than others in a two-country model.

Predictions from a multi-country model look different: the majority of countries alter their domestic asset positions only by a small margin when risk hedging is turned off in Figure 1. This finding suggests that risk hedging becomes less relevant in a multi-country scenario than in a two-country scenario for asset home bias. The main explanation for this contrast between the two scenarios is that, households construct portfolios with 43 instead of 2 assets with non-perfectly correlated returns. If markets are not complete, this large set of assets offers great risk sharing benefits, which weaken the covariance between macro fundamentals and an individual asset's return relative to other assets'. Moreover, when risk hedging needs arise, households adjust 43 assets simultaneously instead of only the domestic asset in their portfolios.

If we compare the hedge ratio for country $i$'s domestic asset

$$\frac{\text{cov}(\widetilde{wL}_{i/I}, \widetilde{R}_{i/I})}{\sum_{k=1}^{I-1} \text{cov}(\widetilde{R}_{i/I}, \widetilde{R}_{k/I})}, \qquad \frac{\text{cov}(\widetilde{P}_{i/I}, \widetilde{R}_{i/I})}{\sum_{k=1}^{I-1} \text{cov}(\widetilde{R}_{i/I}, \widetilde{R}_{k/I})}, \tag{35}$$

to the hedge ratio in the two-country case (34), the denominator embeds the covariance structure of domestic and all the foreign assets in a multi-country case. Table 1 reports that the median hedge ratios of domestic and foreign assets are quantitatively similar:

$$\begin{aligned} \frac{\text{cov}(\widetilde{wL}_{i/I}, \widetilde{R}_{i/I})}{\sum_{k=1}^{I-1} \text{cov}(\widetilde{R}_{i/I}, \widetilde{R}_{k/I})} &\approx \frac{\text{cov}(\widetilde{wL}_{i/I}, \widetilde{R}_{j/I})}{\sum_{k=1}^{I-1} \text{cov}(\widetilde{R}_{j/I}, \widetilde{R}_{k/I})}, \\ \frac{\text{cov}(\widetilde{P}_{i/I}, \widetilde{R}_{i/I})}{\sum_{k=1}^{I-1} \text{cov}(\widetilde{R}_{i/I}, \widetilde{R}_{k/I})} &\approx \frac{\text{cov}(\widetilde{P}_{i/I}, \widetilde{R}_{j/I})}{\sum_{k=1}^{I-1} \text{cov}(\widetilde{R}_{j/I}, \widetilde{R}_{k/I})}, \qquad \forall j \neq i. \end{aligned} \tag{36}$$

This result is in contrast to the two-country case where the hedge ratios of domestic and foreign assets sum up to zero. Now that all the assets offer similar hedging benefits, any portfolio adjustment driven by risk hedging motives is spread across domestic and $I$-1 foreign assets. Therefore, the influence of risk hedging on asset home bias declines

Table 1: Summary of Asset Home Bias and Risk Hedging

| (I) Domestic Asset Share | | Data | No Labor Income Risk | No RER Risk | |
|---|---|---|---|---|---|
| Two-country | Median | 0.378 | 0.171 | 0.234 | |
| | Std Dev | 0.256 | 0.745 | 0.401 | |
| Multi-country | Median | 0.378 | 0.345 | 0.377 | |
| | Std Dev | 0.256 | 0.267 | 0.267 | |
| (II) Hedge Ratio | | Labor Income Risk | | RER Risk | |
| | | Domestic Asset | Foreign Asset | Domestic Asset | Foreign Asset |
| Two-country | Median | -0.235 | 0.235 | 0.193 | -0.193 |
| Multi-country | Median | 0.403 | 0.318 | -0.030 | -0.008 |

Panel (I) presents the share of domestic assets in countries' portfolios in the data and in the model where labor income and RER risks are turned off. Panel (II) presents the model-predicted hedge ratio of domestic assets and the median hedge ratio of all the foreign assets from each holder country's perspective against labor income and RER risks (36). Results are reported for 1) a multi-country case where there are 43 countries with bilateral trade and financial linkages, and 2) a two-country case where each of the countries in the sample treats itself as the domestic economy and all the other countries in the world as the aggregate foreign economy. See calibration strategies of both settings in Section 2.2, cross-country plots in Figure 1, and complete results by country in Table A.1.

in a multi-country model. Between the two risks, RER risk hedging generates weaker adjustment in domestic asset holdings, supported by its smaller absolute value of hedge ratios reported in Table 1. This happens as goods prices are jointly determined by all the countries in the trade structure where the comovment between RER fluctuations and asset returns becomes significantly weaker. Hence, shutting down RER risk barely affects domestic asset positions.

The prediction from this multi-country model that risk hedging is not essential for asset home bias is supported by the empirical evidence from Massa and Simonov (2006) and van Wincoop and Warnock (2010). They find the correlation between asset returns and labor income or real exchange rate to be too low to rationalize home bias by risk-hedging incentives. On the theoretical front, economists also disagree on the implications of risk hedging for portfolio choice. Coeurdacier and Rey (2013) survey different modeling strategies and find their predictions to be sensitive to parametric assumptions, especially in complete markets with two symmetric countries. Therefore, this paper complements the home bias literature by evaluating the importance of risk hedging mechanism in a more general multi-country framework with incomplete markets.

## 3.3 Bilateral Asset Positions

Another contribution of this framework to the international finance literature lies in its ability to examine drivers for bilateral asset positions beyond asset home bias. The qualitative analysis provided for 28 still applies here to explain the potential impacts of second-moment variables, which reflect bilateral and multilateral covariances across countries, on cross-country portfolio choice. Besides these second moments, financial friction in international markets may also affect global financial allocation.

To examine portfolio choice decided jointly by second moments and financial friction, I use this model to explain the strong correlation between bilateral trade and financial linkages across countries in the data. When calculating trade and financial linkages measured as the mean value of bidirectional shares of goods or assets in each other's total trade volume or portfolio averaged over time for a pair of countries (denoted as $\bar{\pi}$ and $\grave{\alpha}$[18]):

$$\pi_{ij} = \frac{\bar{\pi}_{ij} + \bar{\pi}_{ji}}{2}, \qquad \alpha_{ij} = \frac{\grave{\alpha}_{ij} + \grave{\alpha}_{ji}}{2}, \qquad \forall i, j \in \{1, 2, ..., I\}, \tag{37}$$

I find the correlation between trade ($\pi_{ij}$) and financial ($\alpha_{ij}$) linkages to be as high as 0.825 in the sample of country pairs. This finding that countries which trade more with each other in the goods market also hold more of each others' assets in the financial market verifies the strong connection between the two channels of globalization.

There are two potential explanations for this empirical regularity based on the model. First, trade linkages influence cross-country covariances of macro and financial variables relevant for risk hedging and risk sharing, to the extent that bilateral trade linkages predict financial linkages. Second, cross-country financial and trade frictions are shaped by common factors such as geographic distance, cultural similarity, information accessibility, and regional economic integration, all of which are embedded in the friction matrix $F$.[19] Hence, the high correlation of the two frictions potentially generates synchronous linkages in the two channels. I use this rich structural model to distinguish between these two hypotheses, driven by either second moments or financial friction, to explain the connection between bilateral asset positions and bilateral trade ties.

I will focus on the design of exercises for financial friction and risk sharing. Together with the risk hedging analysis provided earlier, these counterfactual exercises quantify the influence of second moments on observed bilateral asset positions. To examine the degree of risk sharing affected by the existence of financial friction, I set the matrix of

[18] The relationship between bilateral portfolio weights denoted as $\grave{\alpha}_{ij}$ and its theoretical counterpart solved from the model $\check{\alpha}_{ij}$ is described by Equation B.1. In quantitative exercises, I solve for $\check{\alpha}_{ij}^{ctf}$ from the model and then use this equation to covert it to $\grave{\alpha}_{ij}$ with which I compute $\alpha_{ij}$ in 37.

[19] As mentioned in footnote 9, barriers to financial allocation in this model may take various forms including these factors, which influence countries' portfolio choice by shaping financial frictions.

frictions equal to zero in the general portfolio equation (29) and solve for portfolios with

$$\frac{\gamma}{\zeta}E_t[\eta\widetilde{Y}\widetilde{R}' + (\frac{\zeta}{\gamma} - \zeta - s)\widetilde{P}\widetilde{R}' - s\widetilde{I}\widetilde{R}' - \widetilde{D}\widetilde{R}' + \widetilde{Z}\widetilde{R}' + \breve{\alpha}\widetilde{R}\widetilde{R}'] = 0. \tag{38}$$

The counterfactual portfolio derived from this equation absent financial friction (denoted as $\breve{\alpha}^{ctf3}$) will support perfect risk sharing, since the number of nonperfectly correlated assets equals the number of shocks in this model so that markets are locally complete around the steady state (Coeurdacier and Gourinchas (2016)). Therefore, the deviation of counterfactual from observed portfolios measures market incompleteness. To solve for portfolios in locally complete markets from 38, I use the analytical solution from Devereux and Sutherland (2011) in such a case without financial friction (Equation D.12 in Appendix D.1). The derived $\breve{\alpha}^{ctf3}$ also allows me to recover cross-country financial frictions. Specifically, I plug $\breve{\alpha}^{ctf3}$ in 38 and $\breve{\alpha}^{org}$ in 29, and take the difference between these two portfolio equations to quantify the friction matrix $F$:

$$\frac{\gamma}{\zeta}E_t[(\breve{\alpha}^{org} - \breve{\alpha}^{ctf3})\widetilde{R}\widetilde{R}'] = \frac{1}{2}F. \tag{39}$$

If friction exists to cause asset market incompleteness, international trade facilitates risk sharing through price adjustment, a classic mechanism that has been studied in the international macro literature.[20] To test whether this mechanism may affect countries' financial allocation, I shut down the influence of RER variability on cross-country consumption differential and derive portfolio from[21]

$$\frac{\gamma}{\zeta}E_t[\eta\widetilde{Y}\widetilde{R}' + (-\zeta - s)\widetilde{P}\widetilde{R}' - s\widetilde{I}\widetilde{R}' - \widetilde{D}\widetilde{R}' + \widetilde{Z}\widetilde{R}' + \breve{\alpha}^{ctf4}\widetilde{R}\widetilde{R}'] = \frac{1}{2}F. \tag{40}$$

Differences in bilateral financial frictions also potentially drive cross-country asset positions. To turn off heterogeneity of frictions which influence country $i$'s choice among different investment destinations, I set all of its bilateral financial frictions with other countries to be the same as its median level across foreign destinations solved from Equation 39 denoted as $\bar{F}_i$. Therefore, elements in the counterfactual matrix of financial

[20] For example, Obstfeld and Rogoff (2000), Corsetti et al. (2008), and Fitzgerald (2012) suggest that trade affects consumption risk sharing through terms-of-trade (TOT) adjustment in response to supply shocks. This paper contributes to this macro literature by embedding the global trade structure in a multi-country DSGE framework, whose prediction for countries' TOT and RER variability differs from that in a standard two-country setup.

[21] Specifically, I turn off the relative price change across countries which reflects RER adjustment in price-adjusted consumption differential: $\widetilde{P}_{i,t+1} - \widetilde{P}_{I,t+1} = 0$ for $\widetilde{C}^p_{i,t+1} - \widetilde{C}^p_{I,t+1}$ defined in C.9.

frictions $F^{ctf}$ become

$$F^{ctf}(i,j) = \bar{F}_i, \qquad \forall j \neq i \in \{1, 2, ..., I\text{-}1\} \tag{41}$$

in the portfolio equation under homogenous friction for each holder country

$$\frac{\gamma}{\zeta} E_t[\eta \widetilde{Y} \widetilde{R}' + (\frac{\zeta}{\gamma} - \zeta - s)\widetilde{P}\widetilde{R}' - s\widetilde{I}\widetilde{R}' - \widetilde{D}\widetilde{R}' + \widetilde{Z}\widetilde{R}' + \breve{\alpha}^{ctf5}\widetilde{R}\widetilde{R}'] = \frac{1}{2}F^{ctf}. \tag{42}$$

Another important implication of market incompleteness is that, countries need to consider the asset covariance structure when constructing optimal portfolios for risk diversification. As discussed earlier, risk diversification affects portfolio choice in two aspects ((i) and (v) in 28). First, the incentive of risk diversification is driven by households' need to diversify the risk of domestic assets. Second, the pattern of risk diversification is shaped by the asset covariance-variance structure based on which portfolio weights are determined. Therefore, to turn off the influences of risk diversification on financial allocation, first I exclude a country's domestic asset return denoted as $R^H$ from its wealth constraint to shut down the need for risk diversification. Second I shut down the heterogeneity in bilateral asset covariances when solving for bilateral portfolio weights. Specifically, a holder country $i$'s asset covariances with others is set to be the same as their median level across foreign investment destinations denoted as $R\bar{R}_i$, which implies that elements in the counterfactual asset covariance matrix $RR'^{ctf}$ are[22]

$$\widetilde{R}\widetilde{R}'^{ctf}(i,j) = R\bar{R}_i, \qquad \forall j \neq i \in \{1, 2, ..., I\text{-}1\}. \tag{43}$$

Hence, the counterfactual portfolio equation absent risk diversification is

$$\frac{\gamma}{\zeta} E_t[\eta \widetilde{Y} \widetilde{R}' + (\frac{\zeta}{\gamma} - \zeta - s)\widetilde{P}\widetilde{R}' - s\widetilde{I}\widetilde{R}' - \widetilde{D}\widetilde{R}' + \widetilde{Z}\widetilde{R}' - \widetilde{R}^H\widetilde{R}' + \breve{\alpha}^{ctf6}\widetilde{R}\widetilde{R}'^{ctf}] = \frac{1}{2}F. \tag{44}$$

We can further split 44 into two equations, whose difference from the original portfolio equation (29) determines counterfactual portfolios influenced separately by the incentive and the pattern of diversification:

$$\frac{\gamma}{\zeta} E_t[\widetilde{R}^H\widetilde{R}' + (\breve{\alpha}^{org} - \breve{\alpha}^{ctf})\widetilde{R}\widetilde{R}'] = 0, \qquad \frac{\gamma}{\zeta} E_t[\breve{\alpha}^{org}\widetilde{R}\widetilde{R}' - \breve{\alpha}^{ctf}\widetilde{R}\widetilde{R}'^{ctf}] = 0. \tag{45}$$

[22]This exercise focuses on the variation of bilateral covariance between a holder country's domestic asset with all the foreign assets, taking the covariance among foreign assets (multilateral resistance) constant for comparative statics analysis.

Characterized by the portfolio equations above, the shifts of counterfactual from original portfolios quantify the contribution of risk hedging, risk sharing, financial friction, and risk diversification to the variation in countries' observed bilateral asset positions.

Table 2: Bilateral Financial and Trade Linkages

| | Data<br>(1). Eqn 29 | No labor income risk hedging<br>(2). Eqn 30 | No RER risk hedging<br>(3). Eqn 32 |
|---|---|---|---|
| Corr $(\alpha_{ij}, \pi_{ij})$ | 0.825 | 0.527 | 0.816 |
| Std Dev $(\alpha_{ij})$ | 0.076 | 0.079 | 0.078 |
| | Complete markets (no friction)<br>(4). Eqn 38 | No RER risk sharing<br>(5). Eqn 40 | Homogeneous financial friction<br>(6). Eqn 42 |
| Corr $(\alpha_{ij}, \pi_{ij})$ | 0.170 | 0.825 | -0.080 |
| Std Dev $(\alpha_{ij})$ | 0.455 | 0.075 | 0.927 |

This table presents the correlation coefficient between bilateral trade and financial linkages (defined in 37) and the standard deviation of bilateral financial linkages in scenarios where risk hedging is turned off, markets are complete, RER does not influence consumption risk sharing, and each country faces the same financial friction and asset covariance across investment destinations. The corresponding portfolio equations for different scenarios are marked in the table.

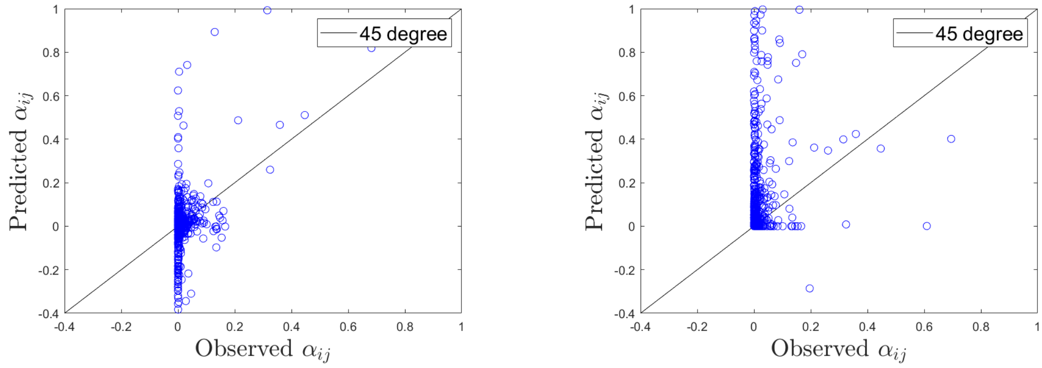
Table 2 summarizes model-predicted bilateral financial linkages ($\alpha_{ij}$ defined in 37) in different scenarios. Since the variable's mean and median values are always close to zero, I focus on second moments including its dispersion measured by standard deviation and its correlation with bilateral trade linkages $\pi_{ij}$. Both moments suggest that asset market incompleteness and heterogeneous financial frictions are major determinants of cross-country financial linkages (columns (4) and (6) in the table). Compared to them, risk hedging is not as important for observed bilateral asset positions (columns (2) and (3)). For example, the correlation between bilateral financial and trade linkages drops from 0.825 in the data to 0.527 and 0.816 when either labor income or RER risk hedging is turned off. The same reasoning for the home bias analysis applies here to explain the negligible effect of RER risk hedging: the covariance between RER fluctuations and individual asset returns is low since price is jointly determined by all the countries in the trade structure. This also explains the result in column (5) where shutting down risk sharing through RER adjustment in the trade channel barely changes the comovement of bilateral financial and bilateral trade linkages.

Compared to risk hedging, financial friction plays a more crucial role. Column (4) suggests that in complete markets where there is no friction, the correlation between trade and financial linkages drops by 79% to 0.17. Hence, countries would shift portfolios substantially from their observed allocations if markets were complete. The huge contrast between original and counterfactual portfolios measures the degree of market incompleteness caused by the existence of cross-country financial frictions. The drastic decline of the correlation between trade and financial linkages is attributable to the fact that bilateral financial frictions are highly correlated with bilateral trade costs. For example, these two frictions covary with common gravity variables such as geographic distance, regional trade agreements, common border, language, and legal system. Therefore, setting all the financial frictions to zero leaves bilateral asset holdings less synchronized with bilateral trade ties, the latter of which is still affected by these gravity variables.

To further establish the importance of the variation beyond the existence of financial frictions for bilateral asset positions, I solve for portfolios by assuming a country's bilateral frictions are the same as their median value across destinations (41). Column (6) of Table 2 reports that the correlation between financial and trade linkages declines even further to a negative value -0.08 in this scenario. The interpretation for this result is that, the assumption of homogeneous financial friction redistributes portfolio weights from investment destinations where financial friction is lower to where the friction is higher. If bilateral financial frictions and trade costs are highly correlated, country-pairs with lower trade costs now face higher financial frictions, and vice versa for country-pairs with higher trade costs which face lower financial frictions. This reasoning explains the negative correlation between bilateral trade and financial linkages under the counterfactual assumption. To illustrate this point, Figure 2 plots the comparison between the scenarios in complete markets (under no friction) and under homogeneous friction. Their main difference is the kink in 2b which represents the portfolio redistribution under homogeneous financial friction. This is different from the pattern in 2a where countries freely decide on portfolio allocations including taking short-sale positions absent financial friction.[23] Since model-predicted financial linkages deviate significantly from observed ones in both scatter plots which look different from each other, financial frictions influence global asset allocation through two distinct mechanisms. First, the variation in bilateral frictions alters the direction of global financial investment. Second, the existence of these frictions causes market incompleteness and impedes international risk sharing. These mechanisms can also explain the strong connection between bilateral financial and trade

[23]The model does not impose a short-sale constraint so asset holdings can be negative. Having the constraint in the model will further complicate the portfolio choice problem by introducing nonlinearity into its computation. Since this paper mainly focuses on comparative statics, the relative change of counterfactual from observed asset positions is more important than their absolute values and signs. From Figure 2, the impact of financial frictions on portfolios is sizable, even though they are assumed to be second-order in magnitude to be consistent with DSTW's solution method. Therefore, these frictions can generate low bilateral portfolio weights consistent with the data.

Figure 2: Bilateral Financial Linkage and Financial Friction




(a) In Complete Markets (Under No Friction) (b) Under Homogeneous Financial Friction

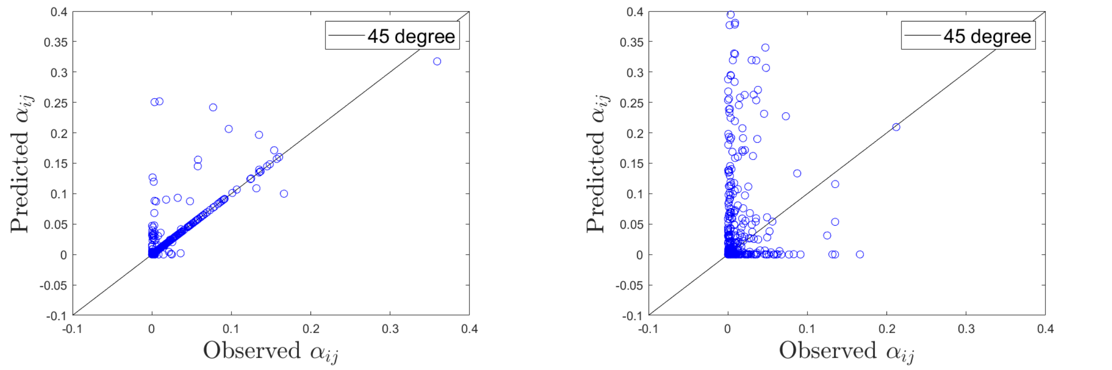
This figure presents bilateral financial linkages (defined in 37) in complete markets with no financial friction and where a holder country's bilateral financial frictions across foreign investment destinations are the same. Observed financial linkages are on the horizontal axis and model-predicted linkages are on the vertical axis.

linkages, which is driven by the high correlation of financial frictions and trade costs.

As risk sharing is not perfect, households need to consider the asset covariance structure to diversify country-specific risks. To investigate the effects of risk diversification on portfolio choice, I examine the incentive and the pattern of diversification separately in terms of their impacts on cross-country financial linkages (Equation 45). Figure 3a shows that if there is no need to diversify the risk of domestic assets, most countries adjust their portfolios only by a negligible magnitude. This can be understood from the fact that a holder country combines its domestic asset with many foreign assets with non-perfectly correlated returns in the portfolio, which makes domestic risk easily diversified away already so that portfolio adjustment is not necessary. However, the weight of each foreign asset in the portfolio is heavily influenced by that asset's covariance with others. In Figure 3b where the heterogeneity in bilateral covariances is shut down, portfolio weights are redistributed such that the share of assets drops for those whose covariance with the domestic asset is low and vice versa. The substantial deviation of counterfactual from observed financial linkages suggests that risk diversification motive is a major determinant of countries' bilateral asset positions: countries' choice of investment destination is strongly directed by the diversification benefit offered by different assets.

To empirically validate the importance of risk diversification and financial friction for portfolios, I test whether their measures covary with observed bilateral asset positions. For risk diversification, I consider cross-country correlation of productivity shocks (see Appendix B for estimation) which are the drivers of risks in the model. For bilateral

Figure 3: Bilateral Financial Linkage and Risk Diversification




(a) No Diversification for Domestic Asset (b) Under Homogeneous Asset Covariance

This figure presents bilateral financial linkages (defined in 37) in the cases where there is no need to diversify the risk associated with domestic asset returns and where a holder country's bilateral asset covariances with assets of different foreign investment destinations are the same. Financial linkages observed in the data are on the horizontal axis and model-predicted linkages are on the vertical axis.

frictions, I use a country-pair's average Chinn-Ito index values to measure their capital account openness. Meanwhile, I obtain geographic distance and other gravity variables from the CEPII dataset. Besides, I include a country pair's bilateral RER as a control variable and get its measure from the ratio of the pair's CPI-based real effective exchange rates from the IMF divided by their ratio of nominal exchange rates.[24]

Table 3 reports the empirical results for the variation of bilateral asset positions. Column (1) shows that bilateral holdings increase in the GDPs of the holder (origin) and asset (destination) countries and decrease in geographic distance, consistent with the gravity model of international finance documented by Portes and Rey (2005). Column (2), by adding origin-, destination-, time-fixed effects and gravity variables, accounts for most of the variability of asset positions in the data implied by the high $R^2$ value. Column (3) further improves the prediction and shows that capital account openness facilitates cross-country investment. Controlling for this institutional feature, stronger productivity comovement reduces bilateral asset positions, as suggested by the negative coefficient for the interaction term of productivity correlation and the Chinn-Ito index in column (4). This finding that portfolio choice is influenced by risk diversification echoes Coeurdacier and Guibaud (2011) and Bergin and Pyun (2016) who empirically and theoretically

[24]Adjusting variables for nominal exchange rate is a common empirical strategy in the home bias literature (for example Coeurdacier and Gourinchas (2016)). The purpose is to control for nominal exchange rate fluctuations in the data which cannot be explained by macro fundamentals in an open economy macro model (also known as the exchange rate disconnect puzzle).

Table 3: Sources of Variation for Bilateral Asset Positions

| Dep. Var: log(Bilateral Holdings) | ( 1 ) | ( 2 ) | ( 3 ) | ( 4 ) | ( 5 ) |
|---|---|---|---|---|---|
| log(GDP$_o$) | 1.245 *** | 1.108 *** | 1.085 *** | 1.111 *** | 1.215 *** |
| | ( 0.034 ) | ( 0.061 ) | ( 0.062 ) | ( 0.059 ) | ( 0.065 ) |
| log(GDP$_d$) | 1.442 *** | -0.012 | 0.042 | 0.112 | 0.196 * |
| | ( 0.032 ) | ( 0.093 ) | ( 0.094 ) | ( 0.089 ) | ( 0.101 ) |
| log(dist) | -0.709 *** | -1.167 *** | -1.202 *** | -1.033 *** | -1.076 *** |
| | ( 0.037 ) | ( 0.021 ) | ( 0.022 ) | ( 0.022 ) | ( 0.025 ) |
| Chinn-Ito | | | 0.674 ** | 2.288 *** | 3.385 *** |
| | | | ( 0.298 ) | ( 0.293 ) | ( 0.918 ) |
| corr(T) | | | | 5.049 *** | 5.230 *** |
| | | | | ( 0.253 ) | ( 0.268 ) |
| Chinn × corr(T) | | | | -4.501 *** | -4.747 *** |
| | | | | ( 0.269 ) | ( 0.284 ) |
| RER | | | | | 0.512 |
| | | | | | ( 0.689 ) |
| Chinn × RER | | | | | -0.975 |
| | | | | | ( 0.855 ) |
| Fixed Effects | N | Y | Y | Y | Y |
| Gravity Variables | N | Y | Y | Y | Y |
| Observations | 22,448 | 22,448 | 20,807 | 20,807 | 17,105 |
| Adjusted $R^2$ | 0.123 | 0.957 | 0.959 | 0.961 | 0.965 |

Robust standard errors in parentheses.***significant at 1%, ** significant at 5%, and * significant at 10%. The dependent variable is bilateral asset holdings in logs from Factset/Lionshare. GDP$_o$ and GDP$_d$ are the GDPs of the investment origin (holder) and destination (asset) country. dist is the population-weighted distance between countries from CEPII. Chinn-Ito is a capital account openness measure here averaged over the country pair. corr(T) is estimated bilateral correlation of productivity shocks (see Appendix B for estimation). Fixed Effects include origin-, destination-, and time-FE. Gravity variables include CEPII's dummy variables for contiguity, regional trade agreements, common official language, religion, and legal origin. RER is the ratio of origin to destination country's real exchange rate, computed as their IMF's REER divided by nominal exchange rate ratios.

establish the importance of cross-country covariance structure for asset positions. Last, column (5) considers RER but does not find it to be an important determinant of bilateral holdings, consistent with the theoretical result that RER risk hedging is not as essential in this multi-country setting. Therefore, these findings provide empirical support for the predictions from the model about determinants of global financial allocation.

# 4 Conclusion

This paper examines portfolio choice in a DSGE model with 43 countries linked through trade and financial channels. I provide comparative statics analyses with the structural model where I quantify the implications of potential mechanisms for asset allocation, including risk hedging, risk sharing, risk diversification, and financial friction.

The model predicts that hedging of labor income or RER risk becomes less important in a multi-country setting for domestic asset holdings. Furthermore, the model suggests that variations in financial friction and asset covariance are major determinants of observed bilateral asset positions across countries. These general equilibrium analyses provide new insights into cross-country financial allocation.

To obtain these numerical predictions from the model, I develop an approach that simplifies the solution method for a multi-country macro model. In particular, I embed a Ricardian trade model in an open economy DSGE framework. The trade model efficiently characterizes intra-temporal allocation across many countries, which allows me to reduce the dynamic system when solving inter-temporal allocation with the linearization method for DSGE frameworks. Beyond portfolio choice, this approach which combines trade and macro methods has many other applications in open economy macro. We can follow the approach to develop international macro models with many countries linked through frictional financial and trade channels, to explore questions such as the transmission of monetary policy, optimal currency invoicing, and financial allocation during major events such as global liquidity crunch and trade wars. The predictions from this multi-country model, by capturing both bilateral and multilateral comovements across many economies, are richer than those from a small open economy or a two country model.

This paper uses the local linearization method developed by DSTW to derive portfolio choice around a deterministic steady state. Their method offers a powerful yet tractable toolkit widely applicable to DSGE models. The derived solution is close to exact around the point of approximation, but it is less accurate when there are large deviations from the steady state or when the problem exhibits strong non-linearity (see a discussion by Coeurdacier and Rey (2013)). Therefore, if global solution methods (such as policy or value function iterations) for the portfolio choice problem with comparable applicability and tractability become available in the future, financial allocation can be endogenously determined in more general economic environments. Important questions including those related to sovereign defaults and disaster risks can be answered with the development of such solution techniques.

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# Appendices

## A Table

## B Data and Calibration

This section describes the data sources and calibration strategies. The sample of economies includes 43 countries (listed in Table B.1) and the rest of the world (ROW). The time-averaged values over the sample period from 2001-2021 will be used as the steady-state values of variables, including countries' output obtained from the Penn World Table (PWT) and trade balance from the World Bank. Countries' wealth as shares of GDP is recovered from Equation 24. ROW's wealth is calibrated to satisfy the world resource constraint. The values of ROW's macro variables, including output and expenditure, are the difference between the world aggregate values and sample countries' total values of these variables. ROW is treated as the numeraire country in quantitative analyses.

## Table A.1: Asset Home Bias and Risk Hedging

| | | (I) Domestic Asset Share | | | | (II) Hedge Ratio | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Country | Data | No Labor Income Risk | | No RER Risk | | Labor Income Risk | | | | RER Risk | | | |
| | | | | | | Domestic Asset | | Foreign Asset | | Domestic Asset | | Foreign Asset | |
| | | multi-co | two-co | multi-co | two-co | multi-co | two-co | multi-co | two-co | multi-co | two-co | multi-co | two-co |
| AUS | 0.61 | 0.43 | -0.69 | 0.61 | 0.50 | -0.03 | 3.11 | -1.23 | -3.11 | 0.12 | -1.10 | 0.58 | 1.10 |
| AUT | 0.10 | 0.11 | 0.17 | 0.10 | 0.00 | 0.47 | -0.30 | 0.43 | 0.30 | -0.04 | 0.22 | -0.04 | -0.22 |
| BHR | 0.81 | 0.81 | 0.70 | 0.81 | 0.78 | 0.43 | 1.19 | -0.19 | -1.19 | 0.21 | 0.89 | -0.17 | -0.89 |
| BEL | 0.11 | 0.12 | 0.17 | 0.12 | -0.02 | 0.48 | -0.23 | 0.44 | 0.23 | 0.14 | 0.21 | 0.04 | -0.21 |
| BRA | 0.70 | 0.54 | -0.51 | 0.69 | 0.54 | 0.36 | 3.45 | -0.40 | -3.45 | -0.03 | -1.26 | 0.45 | 1.26 |
| CAN | 0.45 | 0.38 | -0.03 | 0.45 | 0.31 | 0.43 | 1.48 | 0.23 | -1.48 | -0.02 | -0.68 | 0.03 | 0.68 |
| CHL | 0.72 | 0.74 | 0.81 | 0.72 | 0.62 | 0.41 | -0.41 | 0.00 | 0.41 | -0.05 | 0.27 | 0.10 | -0.27 |
| CHN | 0.69 | 0.62 | 1.04 | 0.69 | 0.62 | 0.43 | -1.55 | -0.85 | 1.55 | -0.14 | -5.31 | 0.23 | 5.31 |
| CZE | 0.19 | 0.21 | 0.25 | 0.19 | 0.10 | 0.28 | -0.25 | 0.25 | 0.25 | -0.08 | 0.20 | -0.06 | -0.20 |
| DNK | 0.13 | 0.14 | 0.17 | 0.13 | 0.05 | 0.43 | -0.23 | 0.42 | 0.23 | 0.00 | 0.22 | 0.00 | -0.22 |
| FIN | 0.45 | 0.47 | 0.53 | 0.45 | 0.33 | 0.39 | -0.31 | 0.36 | 0.31 | -0.01 | 0.22 | 0.00 | -0.22 |
| FRA | 0.35 | 0.28 | -0.26 | 0.34 | 0.18 | 0.37 | 1.56 | 0.32 | -1.56 | -0.01 | -0.44 | -0.01 | 0.44 |
| DEU | 0.20 | 0.18 | -0.03 | 0.20 | 0.12 | 0.38 | 1.27 | 0.37 | -1.27 | -0.01 | -0.50 | -0.01 | 0.50 |
| GRC | 0.36 | 0.04 | -0.88 | 0.41 | 1.51 | 0.58 | -0.53 | 6.00 | 0.53 | -0.17 | 0.26 | -3.33 | -0.26 |
| HKG | 0.17 | 0.19 | 0.25 | 0.17 | 0.11 | 0.43 | -0.42 | 1.11 | 0.42 | -0.15 | 0.29 | -0.39 | -0.29 |
| HUN | 0.35 | 0.36 | 0.41 | 0.37 | 0.23 | 0.25 | -0.25 | 0.27 | 0.25 | -0.04 | 0.19 | -0.03 | -0.19 |
| IRL | 0.16 | 0.16 | 0.10 | 0.17 | 0.11 | 0.28 | 0.68 | 0.18 | -0.68 | -0.47 | 0.66 | -0.04 | -0.66 |
| ISR | 0.68 | 0.71 | 0.80 | 0.68 | 0.52 | 0.45 | -0.38 | 0.22 | 0.38 | -0.03 | 0.23 | 0.03 | -0.23 |
| ITA | 0.20 | 0.07 | -0.40 | 0.20 | 0.07 | 0.38 | 1.98 | 0.28 | -1.98 | -0.01 | -1.30 | -0.01 | 1.30 |
| JPN | 0.49 | 0.37 | -0.46 | 0.48 | 0.34 | 0.44 | 2.80 | 0.40 | -2.80 | -0.05 | -0.99 | -0.04 | 0.99 |
| KOR | 0.73 | 0.69 | 0.65 | 0.73 | 0.65 | -0.10 | 1.56 | -0.30 | -1.56 | 0.10 | -0.67 | 0.21 | 0.67 |
| KWT | 0.22 | 0.22 | 0.27 | 0.22 | 0.19 | 0.18 | -0.70 | 0.12 | 0.70 | -0.04 | 0.03 | -0.03 | -0.03 |
| LUX | 0.01 | 0.01 | 0.04 | 0.01 | -0.02 | 0.48 | -0.58 | 0.40 | 0.58 | -0.11 | 0.05 | -0.12 | -0.05 |
| MYS | 0.80 | 0.80 | 0.78 | 0.80 | 0.76 | 0.29 | 0.27 | 0.11 | -0.27 | 0.04 | 0.44 | 0.01 | -0.44 |
| MEX | 0.74 | 0.57 | -0.12 | 0.72 | 0.47 | 0.48 | 1.73 | 1.44 | -1.73 | -0.17 | -0.88 | -0.56 | 0.88 |
| NLD | 0.09 | 0.09 | 0.11 | 0.09 | 0.02 | 0.41 | -0.17 | 0.40 | 0.17 | 0.05 | 0.25 | 0.03 | -0.25 |
| NZL | 0.48 | 0.50 | 0.58 | 0.47 | 0.34 | 0.38 | -0.33 | 0.36 | 0.33 | -0.02 | 0.21 | -0.01 | -0.21 |
| NOR | 0.06 | 0.07 | 0.08 | 0.06 | 0.01 | 0.39 | -0.13 | 0.36 | 0.13 | -0.02 | 0.30 | -0.02 | -0.30 |
| PHL | 0.47 | 0.74 | 1.84 | 0.37 | -0.14 | 0.67 | -1.34 | 26.00 | 1.34 | -0.23 | 0.59 | -9.00 | -0.59 |
| POL | 0.76 | 1.12 | 0.90 | 0.75 | 0.56 | 0.28 | -11.54 | 0.16 | 11.54 | -0.06 | 4.62 | 0.00 | -4.62 |
| PRT | 0.56 | 0.71 | 1.14 | 0.53 | -0.18 | 0.39 | -0.41 | 0.27 | 0.41 | 0.00 | 0.22 | 0.05 | -0.22 |
| QAT | 0.18 | 0.18 | 0.21 | 0.18 | 0.18 | 0.15 | -0.61 | 0.13 | 0.61 | -0.05 | 0.07 | -0.05 | -0.07 |
| ROU | 0.82 | 0.33 | -1.10 | 1.11 | 1.96 | 0.50 | -0.48 | 0.57 | 0.48 | -0.17 | 0.24 | -0.22 | -0.24 |
| RUS | 0.83 | 0.81 | 0.39 | 0.83 | 0.77 | 0.18 | 3.34 | 0.05 | -3.34 | -0.06 | -1.20 | 0.00 | 1.20 |
| SGP | 0.11 | 0.10 | 0.17 | 0.11 | 0.07 | 0.48 | -0.93 | 0.34 | 0.93 | -0.15 | 0.09 | -0.10 | -0.09 |
| SVN | 0.62 | 0.64 | 0.66 | 0.63 | 0.51 | 0.23 | -0.24 | 0.23 | 0.24 | -0.01 | 0.19 | -0.01 | -0.19 |
| ESP | 0.30 | 0.20 | -0.27 | 0.30 | 0.13 | 0.36 | 1.50 | 0.33 | -1.50 | 0.01 | -0.75 | 0.00 | 0.75 |
| SWE | 0.38 | 0.39 | 0.45 | 0.38 | 0.29 | 0.43 | -0.35 | 0.41 | 0.35 | -0.01 | 0.26 | -0.01 | -0.26 |
| CHE | 0.16 | 0.16 | 0.18 | 0.16 | 0.10 | 0.40 | -0.16 | 0.36 | 0.16 | 0.03 | 0.26 | 0.02 | -0.26 |
| ARE | 0.34 | 0.34 | 0.46 | 0.34 | 0.31 | 0.35 | -1.46 | 0.07 | 1.46 | 0.16 | -0.39 | -0.02 | 0.39 |
| GBR | 0.38 | 0.34 | -0.39 | 0.37 | 0.15 | 0.42 | 1.48 | 0.32 | -1.48 | -0.03 | -0.18 | -0.01 | 0.18 |
| USA | 0.68 | 0.49 | -0.97 | 0.65 | 0.23 | 0.52 | 1.51 | 0.66 | -1.51 | -0.14 | -0.38 | -0.21 | 0.38 |
| ZAF | 0.67 | 0.28 | -2.83 | 0.66 | 0.49 | 0.47 | 10.05 | 0.16 | -10.05 | -0.11 | -3.15 | 0.05 | 3.15 |
| Median | 0.38 | 0.34 | 0.17 | 0.38 | 0.23 | 0.40 | -0.23 | 0.32 | 0.23 | -0.03 | 0.19 | -0.01 | -0.19 |
| Std Dev | 0.26 | 0.27 | 0.74 | 0.27 | 0.40 | 0.14 | 2.69 | 4.03 | 2.69 | 0.11 | 1.30 | 1.46 | 1.30 |

This table presents domestic asset positions and hedge ratios against labor income and real exchange rate (RER) risks. Section (I) reports the share of domestic assets in portfolios in the data and predicted by the model where risk hedging is turned off (derived from portfolio Equations 30 and 32). Section (II) reports the hedge ratios against the two risks (defined in 34) for domestic assets and for assets issued by all the foreign countries whose median values are computed for each holder country. Results are reported for 1) a multi-country case (labeled "multi-co") where there are 43 countries with bilateral trade and financial linkages, and 2) a two-country case (labeled "two-co") with each of the countries in the sample treating itself as the domestic economy and all the other countries in the world as the aggregate foreign economy. Table 1 summarizes the median and standard deviation of these variables and Figure 1 is the scatter plot for predicted versus observed domestic asset holding of sample countries.

Table B.1: List of Sample Countries

| Name | Code | Name | Code | Name | Code | Name | Code |
|---|---|---|---|---|---|---|---|
| Australia | AUS | France | FRA | Luxembourg | LUX | Russia | RUS |
| Austria | AUT | Germany | DEU | Malaysia | MYS | Singapore | SGP |
| Bahrain | BHR | Greece | GRC | Mexico | MEX | Slovenia | SVN |
| Belgium | BEL | Hong Kong | HKG | Netherlands | NLD | Spain | ESP |
| Brazil | BRA | Hungary | HUN | New Zealand | NZL | Sweden | SWE |
| Canada | CAN | Ireland | IRL | Norway | NOR | Switzerland | CHE |
| Chile | CHL | Israel | ISR | Philippines | PHL | U.A.E. | ARE |
| China | CHN | Italy | ITA | Poland | POL | United Kingdom | GBR |
| Czech | CZE | Japan | JPN | Portugal | PRT | United States | USA |
| Denmark | DNK | Korea | KOR | Qatar | QAT | South Africa | ZAF |
| Finland | FIN | Kuwait | KWT | Romania | ROU | | |

## B.1 Bilateral Trade and Financial Shares

Cross-country trade data are obtained from the Direction of Trade Statistics (DOTS) compiled by the IMF. I use the bilateral import (CIF) data to calculate a country's spending on goods imported from other countries. A country's spending on its own goods is computed as the difference between its gross expenditure and total imports, both available from the World Development Indicators (WDI) compiled by the World Bank (WB).

Financial data are from Factset/Lionshare, a comprehensive dataset that provides information on institutional investors' holdings of equities across countries. I describe its details in Hu (2023) and its consistency in terms of portfolio composition with macro-level datasets such as IMF's International Financial Statistics. Factset/Lionshare compiles financial data by investors' origin and destination including for domestic assets, using which I calculate bilateral portfolio weights directly. Ideally, asset ownership should include all forms of capital, such as equity, debt, derivatives, and FDI. However, such comprehensive cross-country financial datasets are scarce, and it takes tremendous efforts to merge datasets covering different types of assets given the lack of universal identifiers for institutional holders. Another popular data source for the purpose is the Coordinated Portfolio Investment Survey (CPIS) and Coordinated Direct Investment Survey (CDIS). I look into these data and find their coverage to be much smaller than Factset/Lionshare's especially for non-OECD countries. Meanwhile, their methodology documentation states that data construction involves much imputation. Such procedure may have caused data anomaly such as negative assets which is difficult to interpret and treat properly, as excluding these values makes the matrix of bilateral portfolio weights even more sparse. For these reasons, I use Factset/Lionshare as the data source to calibrate bilateral portfolio weights, knowing it is not perfect either.

For country $i$'s holding of $j$'s asset, the relationship between its observed portfolio weight from the data denoted as $\grave{\alpha}_{ij}$ (with $\sum_j \grave{\alpha}_{ij} = 1$) and its theoretical counterpart solved from the model (defined in Equations 18 and C.11) $\check{\alpha}_{ij}$ (with $\sum_j \check{\alpha}_{ij} = \frac{\check{D}_i}{\bar{\beta}}$) is

$$\check{\alpha}_{ij} = \frac{\grave{\alpha}_{ij}}{\bar{\beta}}(\check{D}_i + \frac{\bar{q}_i}{\bar{Y}_i}) - \mathbb{1}(i = j)\frac{1}{\bar{\beta}}\frac{\bar{q}_i}{\bar{Y}_i}, \qquad \forall i, j \in \{1, 2, ..., I\} \tag{B.1}$$

where $\frac{\bar{q}_i}{\bar{Y}_i}$ is the equilibrium ratio of asset value to output based on the parameters in the model. This term is included since $\check{\alpha}_{ii}$ is defined as a net holding in 18, adding a unit share to which multiplied by the asset value yields the nominal value of domestic asset holding, consistent with the way portfolio weights are calculated in the data.

As the analysis in this paper covers two channels, the sample of countries includes those with both trade and financial data available (see Table B.1). Factset/Lionshare's coverage expanded significantly several years after its initial launch in 1998, therefore I choose 2001-2021 as the sample period over which I use time-averaged trade and portfolio shares to calibrate bilateral trade and financial linkages across countries respectively.

## B.2 Productivity

The estimation of productivity consistent with the Eaton and Kortum (2002) model is modified from the approach developed by Levchenko and Zhang (2014), who infer Ricardian productivity from bilateral trade data.

Let country $i$'s production cost be denoted as

$$c_{i,t} = (r_{i,t}^{\mu} w_{i,t}^{1-\mu})^{\eta} P_{i,t}^{1-\eta}. \tag{B.2}$$

It follows from Equation 9 that trade shares for any destination country $j$ should satisfy

$$\frac{\pi_{ij,t}}{\pi_{jj,t}} = \frac{T_{i,t}}{T_{j,t}}(\frac{\tau_{ij,t}c_{i,t}}{c_{j,t}})^{-\theta}. \tag{B.3}$$

As the left hand side is directly observable from the trade data, we can recover relative productivity $\frac{T_{i,t}}{T_{j,t}}$ after estimating bilateral trade friction $\tau_{ij,t}$ and relative input cost $\frac{c_{i,t}}{c_{j,t}}$.

I follow the trade literature by estimating bilateral trade costs $\hat{\tau}_{ij,t}$ from a combination of gravity variables including geographic distance divided into intervals set by Eaton and Kortum (2002), dummies for contiguity, common language, common colonizer, common religion, common legal system, and regional trade agreements. These gravity variables are sourced from the CEPII.

I estimate a country's production cost (denoted as $\hat{c}_{i,t}$) based on the information from the PWT. Specifically, I compute a country's wage ($w$) as the ratio of its total labor compensation (output-side GDP ($rgdpo$) $\times$ share of labor compensation in GDP ($labsh$)) to total labor hours (number of employees ($emp$) $\times$ average hours per employee ($avc$)). Price of domestic absorption ($pl_{da}$) and price of capital services ($pl_k$) are used as the proxies for the price of intermediate inputs and capital rental fee respectively. Besides, I calibrate the share of intermediate input in production $\eta = .312$ based on Dekle et al. (2007) and the share of labor input $1 - \mu$ as country-specific $labsh$ from the PWT. The production cost of ROW is calculated as the median cost across countries not included in Table B.1.

The full estimating specification for all the country pairs in the sample follows

$$\ln(\frac{\pi_{ij,t}}{\pi_{jj,t}}) = \ln(T_{i,t}\hat{c}_{i,t}^{-\theta}) - \ln(T_{j,t}\hat{c}_{j,t}^{-\theta}) - \theta\hat{\tau}_{ij,t} + \gamma_{ij,t}, \tag{B.4}$$

The first two terms on the right $\ln(T_{i,t}\hat{c}_{i,t}^{-\theta})$ and $\ln(T_{j,t}\hat{c}_{j,t}^{-\theta})$ can be captured by the exporter and importer fixed effects respectively when running the estimation. $\hat{\tau}_{ij,t}$ represents the estimated bilateral trade costs as a linear combination of the gravity variables described above and $\gamma_{ij,t}$ stands for error terms. Exponentiating the importer fixed effects yields a term that combines country $j$'s productivity and cost denoted as

$$\widehat{Tc}_{j,t} = T_{j,t}\hat{c}_{j,t}^{-\theta}. \tag{B.5}$$

If the US is the benchmark country whose productivity ($T_{US,t}$) is its TFP value from the PWT ($rtfpna$). Then other countries' Ricardian productivity can be calculated as

$$T_{j,t} = T_{US,t}\frac{\widehat{Tc}_{j,t}}{\widehat{Tc}_{US,t}}(\frac{\hat{c}_{j,t}}{\hat{c}_{US,t}})^{\theta}, \tag{B.6}$$

where trade elasticity $\theta = 4$ following Simonovska and Waugh (2014). After calculating countries' dynamic productivity $T_{j,t}$, I compute its mean value over time $\bar{T}$. Based on the random effects model with all the countries, the persistence parameter in the AR(1) process of productivity is estimated to be 0.85. Using these persistence and mean values in the AR(1) process (7) allows me to recover productivity innovations and estimate their cross-country covariance matrix $\Sigma_T$.

# C Portfolio Analysis

## C.1 Portfolio Choice Problem

This section characterizes the portfolio choice problem in a multi-country framework. To solve the portfolio choice problem with Devereux and Sutherland (2011) and Tille and van Wincoop (2010) (DSTW)'s method, let us assume $I$ is a numeraire country when deriving the world matrix of steady-state portfolio weights[25]

$$\begin{bmatrix} \bar{\alpha}_{11} & \bar{\alpha}_{12} & \cdots & \bar{\alpha}_{1I\text{-}1} \\ \bar{\alpha}_{21} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \vdots & & & \bar{\alpha}_{I\text{-}2I\text{-}1} \\ \bar{\alpha}_{I\text{-}11} & \cdots & \bar{\alpha}_{I\text{-}1I\text{-}2} & \bar{\alpha}_{I\text{-}1I\text{-}1} \end{bmatrix} \tag{C.1}$$

whose elements in the $i^{th}$ row are decided by country $i$'s Euler equation (15) which satisfies

$$E_t[\frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}} e^{-f_{i1}} R_{1,t+1}] = ... = E_t[\frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}} e^{-f_{iI\text{-}1}} R_{I\text{-}1,t+1}] = E_t[\frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}} e^{-f_{iI}} R_{I,t+1}]. \tag{C.2}$$

Portfolios are derived from the second-order Taylor expansion of Equation C.2 while taking the difference in returns between the numeraire asset $I$ and all the other assets:

$$E_t[\widetilde{R}_{x,t+1} + \frac{1}{2}\widetilde{R}_{x,t+1}^2 - \widetilde{R}_{x,t+1}(\gamma\widetilde{C}_{i,t+1} + \widetilde{P}_{i,t+1})] = -\frac{1}{2}F_i + \mathcal{O}(\epsilon^3), \tag{C.3}$$

where a tilde represents the log-deviation of any variable from its steady state marked with a bar:

$$\widetilde{A}_t = \ln(\frac{A_t - \bar{A}}{\bar{A}}). \tag{C.4}$$

$R_{x,t+1}$ denotes a vector of excess returns relative to the numeraire asset

$$\widetilde{R}'_{x,t+1} = [\widetilde{R}_{1,t+1} - \widetilde{R}_{I,t+1}, \widetilde{R}_{2,t+1} - \widetilde{R}_{I,t+1}, ..., \widetilde{R}_{I\text{-}1,t+1} - \widetilde{R}_{I,t+1}], \tag{C.5}$$

$R_{x,t+1}^2$ denotes the vector of excess squared returns

$$\widetilde{R}_{x,t+1}^{2\prime} = [\widetilde{R}_{1,t+1}^2 - \widetilde{R}_{I,t+1}^2, \widetilde{R}_{2,t+1}^2 - \widetilde{R}_{I,t+1}^2, ..., \widetilde{R}_{I-1,t+1}^2 - \widetilde{R}_{I,t+1}^2], \tag{C.6}$$

and $F_i$ denotes $i$'s vector of financial frictions defined as

$$F'_i = [f_{iI} - f_{i1}, f_{iI} - f_{i2}, ..., f_{iI} - f_{iI\text{-}1}], \tag{C.7}$$

whose $k^{th}$ element represents the additional financial friction country $i$'s households incur when holding $I$'s relative to $k$'s asset. $\mathcal{O}(\epsilon^3)$ captures all terms of order higher than two.

The difference between any country $i$'s and the numeraire country $I$'s expanded Euler

[25] The portfolio matrix's dimension is $(I\text{-}1) \times (I\text{-}1)$ instead of $I \times I$. For the remaining assets positions, country $i$'s holding of the numeraire asset is decided by the difference between its aggregate asset position and its bilateral holding of other assets. Meanwhile, numeraire country $I$'s holding of any asset $j$ is decided by $j$'s market clearing condition that the supply of the asset equals the demand.

equations (C.3) is

$$E_t[\widetilde{C}^p_{i/I,t+1}\widetilde{R}'_{x,t+1}] = E_t[(\widetilde{C}^p_{i,t+1} - \widetilde{C}^p_{I,t+1})\widetilde{R}'_{x,t+1}] = \frac{1}{2}F_{iI} + \mathcal{O}(\epsilon^3), \qquad \forall i \in [1, I\text{-}1], \quad \text{(C.8)}$$

where $C^p_{i,t+1}$ is $i$'s price-adjusted marginal utility of consumption ($C^p_{i,t+1} = P_{i,t+1}/C^{-\gamma}_{i,t+1}$) and $\widetilde{C}^p_{i/I,t+1}$ is the price- and utility-adjusted consumption differential

$$\widetilde{C}^p_{i/I,t+1} = \widetilde{C}^p_{i,t+1} - \widetilde{C}^p_{I,t+1} = \gamma(\widetilde{C}_{i,t+1} - \widetilde{C}_{I,t+1}) + (\widetilde{P}_{i,t+1} - \widetilde{P}_{I,t+1}), \quad \text{(C.9)}$$

while $F_{iI}$ denotes the excess financial frictions faced by $i$ relative to by $I$

$$F_{iI} = F'_i - F'_I. \quad \text{(C.10)}$$

Equation C.8 is country $i$'s portfolio determination equation: the variables on its left covary with $i$'s relative asset positions which are also influenced by financial frictions $F_{iI}$ on the right. Let $\breve{\alpha}_i$ be a vector of the equilibrium portfolio adjusted for $i$'s output and discount factor

$$\breve{\alpha}_i = [\breve{\alpha}_{i1}, \breve{\alpha}_{i2}, ..., \breve{\alpha}_{iI\text{-}1}] = \frac{1}{\bar{\beta}\bar{Y}_i}[\bar{\alpha}_{i1}, \bar{\alpha}_{i2}, ..., \bar{\alpha}_{iI\text{-}1}], \quad \text{(C.11)}$$

$\xi_{i,t+1} = \breve{\alpha}_i\widetilde{R}_{x,t+1}$, which is the weighted sum of individual assets' excess returns, will be the excess portfolio return that adds to $i$'s wealth.

Equation C.8, if stacked vertically with each row representing a holder country, constructs a system of equations for the world bilateral portfolio matrix (C.1) to be solved. If $\widetilde{C}^p_{x,t+1}$ is the vector of all the countries' consumption differential (defined in C.9) relative to the numeraire country $I$

$$\widetilde{C}^{p\prime}_{x,t+1} = [\widetilde{C}^p_{1,t+1} - \widetilde{C}^p_{I,t+1}, \widetilde{C}^p_{2,t+1} - \widetilde{C}^p_{I,t+1}, ..., \widetilde{C}^p_{I-1,t+1} - \widetilde{C}^p_{I,t+1}], \quad \text{(C.12)}$$

$F$ is the vector of countries' relative financial frictions

$$F^{'} = [F_{1I}, F_{2I}, ..., F_{I-1I}], \quad \text{(C.13)}$$

then the world portfolio matrix consisting of countries' relative asset holdings

$$\breve{\alpha}^{'} = [\breve{\alpha}_1, \breve{\alpha}_2, ..., \breve{\alpha}_{I-1}] - \breve{\alpha}_I \quad \text{(C.14)}$$

can be obtained from the system of portfolio determination equations summarized by

$$E_t(\widetilde{C}^p_{x,t+1}\widetilde{R}'_{x,t+1}) = \frac{1}{2}F + \mathcal{O}(\epsilon^3). \quad \text{(C.15)}$$

## C.2 Qualitative Analysis

This section shows how second-moment variables influence global financial allocation by deriving Equation 27 in Section 3.1.

I start by re-writing the portfolio determination equation C.8 with price- and utility-adjusted consumption of countries $i$ and $I$ (defined in C.9) obtained from their wealth constraints 21:

$$E_t(\widetilde{C}^p_{i,t+1}) = \frac{\gamma}{\zeta_i} E_t[\eta \widetilde{Y}_{i,t+1} + (\frac{\zeta_i}{\gamma} - \zeta_i - s)\widetilde{P}_{i,t+1} - s\widetilde{IV}_{i,t+1} - \widetilde{D}_{i,t+1} + \frac{\widetilde{D}_{i,t}}{\bar{\beta}} + \breve{\alpha}_i \widetilde{R}_{x,t+1}], \quad \text{(C.16)}$$

$$E_t(\widetilde{C}^p_{I,t+1}) = \frac{\gamma}{\zeta_I} E_t[\eta \widetilde{Y}_{I,t+1} + (\frac{\zeta_I}{\gamma} - \zeta_I - s)\widetilde{P}_{I,t+1} - s\widetilde{IV}_{I,t+1} - \widetilde{D}_{I,t+1} + \frac{\widetilde{D}_{I,t}}{\bar{\beta}} + \breve{\alpha}_I \widetilde{R}_{x,t+1}], \quad \text{(C.17)}$$

where $s$ and $\zeta_i$ are countries' equilibrium investment-to-output and consumption-to-output ratios respectively. The consumption differential between $i$ and $I$ is given by the difference between Equations C.16 and C.17:

$$E_t(\widetilde{C}^p_{i/I,t+1}) = \frac{\gamma}{\zeta_i} E_t[\eta \widetilde{Y}_{i/I,t+1} + (\frac{\zeta_i}{\gamma} - \zeta_i - s)\widetilde{P}_{i/I,t+1} - s\widetilde{IV}_{i/I,t+1} - \widetilde{D}^{\Delta}_{i/I,t+1} + \widetilde{Z}_{Ii,t+1} + (\breve{\alpha}_i - \breve{\alpha}_I)\widetilde{R}_{x,t+1}], \quad \text{(C.18)}$$

where $D^{\Delta}_{i/I,t+1}$ denotes the evolution of relative wealth normalized by countries' income:

$$\widetilde{D}^{\Delta}_{i/I,t+1} = \widetilde{D}_{i/I,t+1} - \frac{\widetilde{D}_{i/I,t}}{\bar{\beta}}, \quad \text{with} \quad \widetilde{D}_{i/I,t} = \ln(\frac{D_{i,t} - \bar{D}_i}{\bar{Y}_i}) - \ln(\frac{D_{I,t} - \bar{D}_I}{\bar{Y}_I}). \quad \text{(C.19)}$$

$Z_{Ii,t+1}$ represents the numeraire country $I$'s excess consumption expenditure due to countries' differences in equilibrium consumption-to-output ratios:

$$\widetilde{Z}_{Ii,t+1} = (\zeta_I - \zeta_i)(\widetilde{P}_{I,t+1} + \widetilde{C}_{I,t+1}), \quad \text{with} \quad \zeta_I = \eta - s - \check{D}_I(1 - 1/\bar{\beta}). \quad \text{(C.20)}$$

Country $i$'s portfolio equation (C.8), given the consumption differential (C.18), becomes

$$\begin{aligned} E_t(\widetilde{C}^p_{i/I,t+1}\widetilde{R}'_{x,t+1}) = \frac{\gamma}{\zeta_i} E_t[(\eta \widetilde{Y}_{i/I,t+1} + (\frac{\zeta_i}{\gamma} - \zeta_i - s)\widetilde{P}_{i/I,t+1} \\ - s\widetilde{IV}_{i/I,t+1} - \widetilde{D}^{\Delta}_{i/I,t+1} + \widetilde{Z}_{Ii,t+1} + (\breve{\alpha}_i - \breve{\alpha}_I)\widetilde{R}_{x,t+1})\widetilde{R}'_{x,t+1}] = \frac{1}{2}F_{iI}. \end{aligned} \quad \text{(C.21)}$$

From this equation, $i$'s portfolio is influenced by its second-moment variables:

$$\widetilde{Y}_{i/I,t+1}\widetilde{R}'_{x,t+1}, \; \widetilde{P}_{i/I,t+1}\widetilde{R}'_{x,t+1}, \; \widetilde{IV}_{i/I,t+1}\widetilde{R}'_{x,t+1}, \; \widetilde{D}^{\Delta}_{i/I,t+1}\widetilde{R}'_{x,t+1}, \; \widetilde{R}_{x,t+1}\widetilde{R}'_{x,t+1}, \quad \text{(C.22)}$$

which capture the covariances between excess asset returns $\widetilde{R}_{x,t+1}$ and $i$'s relative to $I$'s macro variables including income ($\widetilde{Y}_{i/I,t+1}$), price ($\widetilde{P}_{i/I,t+1}$), investment ($\widetilde{IV}_{i/I,t+1}$), wealth change ($\widetilde{D}^{\Delta}_{i/I,t+1}$), and the covariance-variance structure of asset returns ($\widetilde{R}_{x,t+1}\widetilde{R}'_{x,t+1}$)

respectively. Stacking C.21 vertically with each row representing a holder country, I express the portfolio equation of all the countries (C.15) with subscripts omitted for simplicity as

$$\frac{\gamma}{\zeta}E_t[\eta\widetilde{Y}\widetilde{R}' + (\frac{\zeta}{\gamma} - \zeta - s)\widetilde{P}\widetilde{R}' - s\widetilde{I}\widetilde{R}' - \widetilde{D}\widetilde{R}' + \widetilde{Z}\widetilde{R}' + \breve{\alpha}\widetilde{R}\widetilde{R}'] = \frac{1}{2}F. \tag{C.23}$$

A portfolio weight derived from this equation $\bar{\alpha}_{ij}$ will be the element in the $i^{th}$ row and $j^{th}$ column of the world portfolio matrix (C.1), which is affected by the second-moment variables listed in the equation. For example, $\widetilde{Y}_{i/I,t+1}\widetilde{R}_{j/I,t+1}$ — the element in the $i^{th}$ row and $j^{th}$ column of $\widetilde{Y}\widetilde{R}'$ which captures the covariance of country $i$'s relative income with $j$'s excess asset return under the shocks in the economy — will influence $i$'s portfolio including its holding of $j$'s asset $\bar{\alpha}_{ij}$. An asset holding is also affected by bilateral financial friction in the friction matrix $F$ whose element in the $i^{th}$ row $j^{th}$ column is

$$F(i,j) = f_{iI} - f_{ij} - f_{II} + f_{Ij}. \tag{C.24}$$

A higher bilateral friction $f_{ij}$, ceteris paribus, lowers bilateral asset holding $\bar{\alpha}_{ij}$. Section 3 provides detailed qualitative and quantitative analyses for the influences of second moments and financial frictions on global financial allocation.

# D Computation

## D.1 Portfolio Choice Solution Method

This section applies the computation technique developed by Devereux and Sutherland (2011) to a multi-country portfolio choice problem. Equation C.8 is country $i$' portfolio determination equation:

$$E_t[\widetilde{C}^p_{i/I,t+1}\widetilde{R}'_{x,t+1}] = \frac{1}{2}F_{iI} + \mathcal{O}(\epsilon^3). \tag{D.1}$$

On the left hand side of this portfolio equation are two components: $i$'s price- and utility-adjusted consumption differential relative to the numeraire country $I$'s ($\widetilde{C}^p_{i/I,t+1}$ defined in C.9) and the excess asset return ($\widetilde{R}_{x,t+1}$ defined in C.5). We need to express these two components in terms of the productivity innovations in the model

$$\epsilon'_t = [\epsilon_{1,t}, \epsilon_{2,t}, ..., \epsilon_{I,t}], \tag{D.2}$$

whose coefficients as a function of asset positions ($\breve{\alpha}_i$ defined in C.11) need to satisfy Equation D.1. In this process, we need to take into consideration that these components covary with country $i$'s wealth influenced by excess portfolio returns defined earlier as

$\xi_{i,t} = \breve{\alpha}_i \widetilde{R}_{x,t}$ which constitute a vector

$$\xi_t' = [\xi_{1,t}, \xi_{2,t}, ..., \xi_{I,t}]. \tag{D.3}$$

$\xi_t$ can be treated as zero-mean i.i.d. random variables as assets are expected to carry the same returns ex-ante which implies $E_t(\widetilde{R}_{x,t+1}) = 0$. Since $\xi_t$ and $\epsilon_t$ are potentially interdependent, Devereux and Sutherland (2011) suggest a two-step procedure: In the first step, the two components in Equation D.1 (consumption differential and excess returns) are expressed as functions of $\epsilon_t$ and $\xi_t$. In the second step, $\xi_t$ is expressed as a function of $\epsilon_t$ so that the two components can be expressed in terms of $\epsilon_t$ only. Evaluating $\epsilon_t$'s coefficients in Equation D.1, which are functions of $\breve{\alpha}$, will enable us to solve the portfolio choice problem.

To implement the procedure, we write the system of equations C.15 in the first step as

$$E_t(\widetilde{C}^p_{x,t+1} \widetilde{R}'_{x,t+1}) = \frac{1}{2}F + \mathcal{O}(\epsilon^3) \quad \text{with} \tag{D.4}$$

$$\widetilde{C}^p_{x,t+1} = D_1 \xi_{t+1} + D_2 \epsilon_{t+1} + D_3 \widetilde{z}_{t+1} + \mathcal{O}(\epsilon^2), \tag{D.5}$$

$$\widetilde{R}_{x,t+1} = R_1 \xi_{t+1} + R_2 \epsilon_{t+1} + \mathcal{O}(\epsilon^2), \tag{D.6}$$

where $R_1, R_2, D_1, D_2, D_3$ are the coefficient matrices extracted from the first-order conditions of the model. $D_1$, $D_2$, and $D_3$ capture the responses of consumption differential $C^p_{x,t+1}$ to excess portfolio returns ($\xi_{t+1}$), to productivity shocks ($\epsilon_{t+1}$), and to other state variables in the model summarized by $z_{t+1}$ respectively. $R_1$ and $R_2$ capture the responses of excess asset returns $\widetilde{R}_{x,t+1}$ to excess portfolio returns and to productivity shocks:

$$D_1 = \frac{\partial C^p_{x,t+1}}{\partial \xi_{t+1}}, \qquad D_2 = \frac{\partial C^p_{x,t+1}}{\partial \epsilon_{t+1}}, \qquad D_3 = \frac{\partial C^p_{x,t+1}}{\partial z_{t+1}} \qquad R_1 = \frac{\partial R_{x,t+1}}{\partial \xi_{t+1}}, \qquad R_2 = \frac{\partial R_{x,t+1}}{\partial \epsilon_{t+1}}. \tag{D.7}$$

In the second step, we substitute out $\xi_t$ as a function of $\epsilon_t$:

$$\xi_{t+1} = H \epsilon_{t+1}, \qquad \text{where} \qquad H = \frac{\breve{\alpha} R_2}{1 - \breve{\alpha} R_1}, \tag{D.8}$$

in order to express variables in Equation D.4 in terms of $\epsilon_t$ only:

$$\widetilde{C}^p_{x,t+1} = \mathfrak{D} \epsilon_{t+1} + D_3 \widetilde{z}_{t+1} + \mathcal{O}(\epsilon^2), \qquad \text{where} \qquad \mathfrak{D} = D_1 H + D_2, \tag{D.9}$$

$$\widetilde{R}_{x,t+1} = \mathfrak{R} \epsilon_{t+1} + \mathcal{O}(\epsilon^2), \qquad \text{where} \qquad \mathfrak{R} = R_1 H + R_2. \tag{D.10}$$

Once we express these two components separately as functions of $\epsilon_{t+1}$, we multiply them to evaluate Equation D.4 in order to solve for portfolio $\breve{\alpha}$ embedded in $H$:

$$E_t(\widetilde{C}^p_{x,t+1} \widetilde{R}'_{x,t+1}) = \mathfrak{D} \Sigma_T \mathfrak{R}' = (D_1 H + D_2) \Sigma_T (H' R_1' + R_2') = \frac{1}{2}F + \mathcal{O}(\epsilon^3). \tag{D.11}$$

In the case with no financial friction which implies $F = 0$, Devereux and Sutherland

(2011) derive an analytical solution to the portfolio choice problem:

$$\breve{\alpha} = [R_2 \Sigma_T D_2' R_1' - D_1 R_2 \Sigma_T R_2']^{-1} R_2 \Sigma_T D_2'. \tag{D.12}$$

Computing these coefficient matrices $R_1, R_2, D_1, D_2$ typically requires employing the linearization method for the whole DSGE model to predict the first-order dynamics of control and state variables in response to the shocks in the economy. However, it is computationally challenging to implement when there are many countries with numerous variables. Appendix D.2 describes a system reduction method for this macro-trade model.

## D.2 System Reduction Method

To facilitate the computation of a large-scale multi-country DSGE framework, this section combines the system reduction method and Uhlig (1995)'s toolkit of undermined coefficients.[26] The general idea of system reduction is introduced by macro economists including King and Watson (2002), Hernandez (2013), and Sims (2017). I adapt their idea to the international context by embedding a gravity trade model in an open economy macro model. The gravity model characterizes intra-temporal allocation across economies efficiently, so we can reduce the number of variables and equations when characterizing inter-temporal allocation by performing eigen-decomposition of the linear system.

To implement the method, let us first list and group variables in the model

$$S = [C, P, w, r, Y, X, \pi, IV, L, d, R, D, q, K, T]'. \tag{D.13}$$

Each element here represents a $I \times 1$ vector of all the countries' variables (except for $\pi$ which is a $I^2 \times 1$ vector for bilateral trade shares and $D$ which is a $I$-1 $\times$ 1 vector for non-numeraire countries' wealth). Following Sims (2017)'s notes, I divide these variables into four groups: the first with *explicitly* forward-looking (jump) variable $S_1$ which is consumption $C$ in this model

$$S_1 = [C]', \tag{D.14}$$

the second with static (redundant) control variables $S_2$, including price $P$, wage $w$, capital rent $r$, output $Y$, expenditure $X$, bilateral trade share $\pi$, capital investment $IV$, labor supply $L$, dividend $d$, and asset return $R$:

$$S_2 = [P, w, r, Y, X, \pi, IV, L, d, R]'. \tag{D.15}$$

As noted by Sims (2017), many of these 'static' control variables in $S_2$ are *implicitly* forward-looking through their dependence of jump variables $S_1$. For example, capital

[26] The system reduction method is also compatible with other standard linear solution techniques for DSGE models including Blanchard and Kahn (1980), Klein (2000), and Sims (2002). As surveyed by Flotho (2009), the main differences among these methods lie in the way they characterize stable solution manifold, treat expectations in the model, and distinguish between predetermined and nonpredetermined variables. The system reduction I propose in this paper is especially complementary to Uhlig (1995)'s method which cannot accommodate a very large state space.

investment ($IV$) can be obtained from households' budget constraint once the jump variable consumption $C$ is characterized. Therefore, these static control variables are sometimes called 'redundant' variables, because they are linear combinations of jump and state variables. $S_2$ can hence be determined intra-temporally once $S_1$ is decided by households' inter-temporal decisions.

Besides control variables, $S_3$ represents endogenous state variables, including wealth $D$ of $I$-1 non-numeraire countries, asset price $q$ and capital stock $K$ of all $I$ countries. $S_4$ represents exogenous state variables $T$ driven by productivity shocks $\epsilon$, and excess portfolio returns $\xi$ of non-numeraire countries[27]

$$S_3 = [D, q, K]', \qquad S_4 = [T, \xi]'. \tag{D.16}$$

Let $n_i$ denotes the number of variables in $S_i$: $n_1 = I, n_2 = 9I+I^2, n_3 = 3I\text{-}1, n_4 = 2I\text{-}1$.

The main idea of the system reduction method is to substitute out static variables ($S_2$), so that we have a smaller linear system with jump and state variables only when performing eigen-decomposition of the DSGE model. Since the gravity trade model, in the form of Eaton and Kortum (2002)'s framework in this paper, builds trade linkages across many countries in a closed form, expressing static variables in terms of jump and state variables becomes easier than in a standard macro model.[28] From the trade block, we can loglinearize the following equations in Eaton and Kortum (2002)'s model (9-12):[29]

Bilateral trade share
$$\widetilde{\pi}_{ij,t} = \widetilde{T}_{i,t} - \theta[\mu\eta\widetilde{r}_{i,t} + (1-\mu)\eta\widetilde{w}_{i,t} + (1-\eta)\widetilde{P}_{i,t}] + \theta\widetilde{P}_{j,t}, \tag{D.17}$$

Price determination
$$\widetilde{P}_{j,t} = \sum_{i=1}^{I} \bar{\pi}_{ij}[-1/\theta\widetilde{T}_{i,t} + \mu\eta\widetilde{r}_{i,t} + (1-\mu)\eta\widetilde{w}_{i,t} + (1-\eta)\widetilde{P}_{i,t}], \tag{D.18}$$

Expenditure
$$\frac{\bar{X}_j}{\bar{Y}_j}\widetilde{X}_{j,t} = (1-\eta)\widetilde{Y}_{j,t} + \zeta_j(\widetilde{P}_{j,t} + \widetilde{C}_{j,t}) + s(\widetilde{P}_{j,t} + \widetilde{IV}_{j,t}), \tag{D.19}$$

Goods market clearing
$$\bar{Y}_i\widetilde{Y}_{i,t} = \sum_{j=1}^{I} \bar{\pi}_{ij}\bar{X}_j(\widetilde{\pi}_{ij,t} + \widetilde{X}_{j,t}). \tag{D.20}$$

These equations from the gravity trade model efficiently characterize intratemporal allo-

[27] As discussed in Appendix D.1, $\xi$ can be treated as a zero-mean i.i.d. random variable that affects countries' wealth when solving for equilibrium portfolios (Devereux and Sutherland (2011)).

[28] It is straightforward to solve for all the countries' wages and prices, when any country's productivity changes, with the gravity equation in Eaton and Kortum (2002)'s model. Alternatively, other trade frameworks including Armington (1969)'s and Melitz (2003))'s, which also provide theoretical foundations for gravity trade flows, can be embedded in this DSGE model.

[29] Besides directly loglinearing these equations, we can also consider the 'exact hat algebra' technique from the trade literature to first solve the gravity model nonlinearly and then take logs to transform it into the linear form.

cation of many countries' output $Y$, price $P$, and trade shares $\pi$ given productivity $T$, once we know countries' expenditure $X$ and capital investment $IV$ determined by their forward-looking decisions on consumption $C$. Besides these variables from the trade block, other static variables in $S_2$ are determined by

$$\text{Dividend} \qquad \widetilde{d}_{i,t} = \frac{\mu\eta}{\mu\eta - s}\widetilde{Y}_{i,t} - \frac{s}{\mu\eta - s}(\widetilde{P}_{i,t} + \widetilde{IV}_{i,t}), \tag{D.21}$$

$$\text{Asset return} \qquad \widetilde{R}_{i,t} = (1-\bar{\beta})\widetilde{d}_{i,t} + \bar{\beta}\widetilde{q}_{i,t} - \widetilde{q}_{i,t-1}, \tag{D.22}$$

$$\text{Capital rental fee} \qquad \widetilde{Y}_{i,t} = \widetilde{K}_{i,t} + \widetilde{r}_{i,t}, \tag{D.23}$$

$$\text{Capital investment} \qquad \widetilde{K}_{i,t} = (1-\delta)\widetilde{K}_{i,t-1} + \delta\widetilde{IV}_{i,t}, \tag{D.24}$$

$$\text{Wage determination} \qquad \widetilde{Y}_{i,t} = \widetilde{w}_{i,t} + \widetilde{L}_{i,t}, \tag{D.25}$$

$$\text{Labor supply} \qquad \kappa_2\widetilde{L}_{i,t} = -\gamma\widetilde{C}_{i,t} - \widetilde{P}_{i,t} + \widetilde{w}_{i,t}. \tag{D.26}$$

There are altogether $9I + I^2$ equations (D.17-D.26) with which we can characterize $n_2 = 9I + I^2$ static variables ($S_2$) as functions of jump ($S_1$) and state variables ($S_3$).

In the next step, I follow Uhlig (1995) to build the loglinearized model in three blocks with 1) non-expectational equations including those from the gravity trade model (D.27), and 2) expectational equations including the Euler equations (D.28), and 3) an exogenous process for shocks (D.29):

$$\mathcal{A}S_{3t} + \mathcal{B}S_{3t-1} + \mathcal{C}S_{12t} + \mathcal{D}S_{4t} = 0, \tag{D.27}$$

$$E_t[\mathcal{F}S_{3t+1} + \mathcal{G}S_{3t} + \mathcal{H}S_{3t-1} + \mathcal{J}S_{12t+1} + \mathcal{K}S_{12t} + \mathcal{L}S_{4t+1} + \mathcal{M}S_{4t}] = 0, \tag{D.28}$$

$$S_{4t} = \mathcal{N}S_{4t-1} + \epsilon_t. \tag{D.29}$$

where all the coefficient matrices $\mathcal{A} - \mathcal{N}$ are obtained from model loglinearization. $S_{12}$ denotes all the control variables including jump and static variables

$$S_{12} = [S_1, S_2]'. \tag{D.30}$$

To separate these control variables for system reduction, we partition the coefficient matrices in non-expectational equations D.27 as

$$\begin{bmatrix} \mathcal{A}_1 \\ (n_1 \times n_3) \\ \mathcal{A}_2 \\ (n_2 \times n_3) \end{bmatrix} S_{3t} + \begin{bmatrix} \mathcal{B}_1 \\ (n_1 \times n_3) \\ \mathcal{B}_2 \\ (n_2 \times n_3) \end{bmatrix} S_{3t-1} + \begin{bmatrix} \mathcal{C}_{11} & \mathcal{C}_{12} \\ (n_1 \times n_1) & (n_1 \times n_2) \\ \mathcal{C}_{21} & \mathcal{C}_{22} \\ (n_2 \times n_1) & (n_2 \times n_2) \end{bmatrix} \begin{bmatrix} S_{1t} \\ S_{2t} \end{bmatrix} + \begin{bmatrix} \mathcal{D}_1 \\ (n_1 \times n_4) \\ \mathcal{D}_2 \\ (n_2 \times n_4) \end{bmatrix} S_{4t} = 0, \tag{D.31}$$

to represent the variables' joint dynamics

$$\mathcal{A}_1 S_{3t} + \mathcal{B}_1 S_{3t-1} + \mathcal{C}_{11} S_{1t} + \mathcal{C}_{12} S_{2t} + \mathcal{D}_1 S_{4t} = 0, \tag{D.32}$$

$$\mathcal{A}_2 S_{3t} + \mathcal{B}_2 S_{3t-1} + \mathcal{C}_{21} S_{1t} + \mathcal{C}_{22} S_{2t} + \mathcal{D}_2 S_{4t} = 0. \tag{D.33}$$

D.32 contains $n_1 = I$ equations (countries' wealth constraints), and D.33 contains $n_2 = 9I + I^2$ equations (D.17-D.26). In particular, D.33 allows us to express static variables $S_{2t}$ as:

$$S_{2t} = \mathcal{C}_{22}^{-1}(-\mathcal{C}_{21}S_{1t} - \mathcal{A}_2 S_{3t} - \mathcal{B}_2 S_{3t-1} - \mathcal{D}_2 S_{4t}). \tag{D.34}$$

Plugging D.34 in D.32 yields a reduced system of $n_1$ equations with state and jump variables only

$$\underbrace{\left(\mathcal{A}_1 - \mathcal{C}_{12}\mathcal{C}_{22}^{-1}\mathcal{A}_2\right)}_{\mathbb{A}} S_{3t} + \underbrace{\left(\mathcal{B}_1 - \mathcal{C}_{12}\mathcal{C}_{22}^{-1}\mathcal{B}_2\right)}_{\mathbb{B}} S_{3t-1} + \underbrace{\left(\mathcal{C}_{11} - \mathcal{C}_{12}\mathcal{C}_{22}^{-1}\mathcal{C}_{21}\right)}_{\mathbb{C}} S_{1t} + \underbrace{\left(\mathcal{D}_1 - \mathcal{C}_{12}\mathcal{C}_{22}^{-1}\mathcal{D}_2\right)}_{\mathbb{D}} S_{4t} = 0, \tag{D.35}$$

where coefficient matrices below underbraces represent the transformed matrices in the reduced system corresponding to the original ones above underbraces.

To determine jump variables, we examine expectational equations in the model. There are $l = 3I - 1$ expectational equations, which include $I$ countries' Euler equations for physical capital investment, $I$ Euler equations for domestic assets of all the countries, and $I-1$ Euler equations for foreign assets of any one holder country. To perform system reduction, we partition coefficient matrices for these equations (D.28)

$$E_t[\mathcal{F}S_{3t+1} + \mathcal{G}S_{3t} + \mathcal{H}S_{3t-1} + \begin{bmatrix} \mathcal{J}_1 & \mathcal{J}_2 \\ (l \times n_1) & (l \times n_2) \end{bmatrix} \begin{bmatrix} S_{1t+1} \\ S_{2t+1} \end{bmatrix} \tag{D.36}$$

$$+ \begin{bmatrix} \mathcal{K}_1 & \mathcal{K}_2 \\ (l \times n_1) & (l \times n_2) \end{bmatrix} \begin{bmatrix} S_{1t} \\ S_{2t} \end{bmatrix} + \mathcal{L}S_{4t+1} + \mathcal{M}S_{4t}] = 0. \tag{D.37}$$

Given this partition, we can use D.34 to rewrite the equations without static variables

$$E_t[\underbrace{\left(\mathcal{F} - \mathcal{J}_2\mathcal{C}_{22}^{-1}\mathcal{A}_2\right)}_{\mathbb{F}} S_{3t+1} + \underbrace{\left(\mathcal{G} - \mathcal{K}_2\mathcal{C}_{22}^{-1}\mathcal{A}_2 - \mathcal{J}_2\mathcal{C}_{22}^{-1}\mathcal{B}_2\right)}_{\mathbb{G}} S_{3t} + \underbrace{\left(\mathcal{H} - \mathcal{K}_2\mathcal{C}_{22}^{-1}\mathcal{B}_2\right)}_{\mathbb{H}} S_{3t-1}$$

$$+ \underbrace{\left(\mathcal{J}_1 - \mathcal{J}_2\mathcal{C}_{22}^{-1}\mathcal{C}_{21}\right)}_{\mathbb{J}} S_{1t+1} + \underbrace{\left(\mathcal{K}_1 - \mathcal{K}_2\mathcal{C}_{22}^{-1}\mathcal{C}_{21}\right)}_{\mathbb{K}} S_{1t} + \underbrace{\left(\mathcal{L} - \mathcal{J}_2\mathcal{C}_{22}^{-1}\mathcal{D}_2\right)}_{\mathbb{L}} S_{4t+1} + \underbrace{\left(\mathcal{M} - \mathcal{K}_2\mathcal{C}_{22}^{-1}\mathcal{D}_2\right)}_{\mathbb{M}} S_{4t}] = 0. \tag{D.38}$$

From D.35 and D.38, we have reduced the original linear system with $n_1+n_2+n_3+n_4 = 14I + I^2 - 1$ variables to one with $n_1 + n_3 + n_4 = 5I - 1$ jump and state variables:

$$\mathbb{A}S_{3t} + \mathbb{B}S_{3t-1} + \mathbb{C}S_{1t} + \mathbb{D}S_{4t} = 0, \tag{D.39}$$

$$E_t[\mathbb{F}S_{3t+1} + \mathbb{G}S_{3t} + \mathbb{H}S_{3t-1} + \mathbb{J}S_{1t+1} + \mathbb{K}S_{1t} + \mathbb{L}S_{4t+1} + \mathbb{M}S_{4t}] = 0. \tag{D.40}$$

$$S_{4t} = \mathbb{N}S_{4t-1} + \epsilon_t. \tag{D.41}$$

In a 43-country case, this procedure saves $n_2 = 2236$ variables (equations) when performing the eigen-decomposition of the linear system. Therefore, this system reduction yields substantial gains in efficiency, with a running time of few minutes on a regular laptop.

When performing eigen-decomposition of the reduced system (D.39-D.41), we follow Uhlig (1995)'s approach below to obtain the recursive equilibrium law of motion for jump and state variables

$$S_{1t} = US_{3t-1} + VS_{4t}, \qquad S_{3t} = PS_{3t-1} + QS_{4t}. \tag{D.42}$$

Specifically, $U$ and $P$ are characterized by the equations

$$\mathbb{C}^0 \mathbb{A} P + \mathbb{C}^0 \mathbb{B} = 0, \tag{D.43}$$

$$(\mathbb{F} - \mathbb{J}\mathbb{C}^+\mathbb{A})P^2 - (\mathbb{J}\mathbb{C}^+\mathbb{B} - \mathbb{G} + \mathbb{K}\mathbb{C}^+\mathbb{A})P - \mathbb{K}\mathbb{C}^+\mathbb{B} + \mathbb{H} = 0, \tag{D.44}$$

$$U = -\mathbb{C}^+(\mathbb{A}P + \mathbb{B}), \tag{D.45}$$

where $\mathbb{C}^+$ is the psudo-inverse of $\mathbb{C}$ and rows of $\mathbb{C}^0$ form a basis of the null space of $\mathbb{C}'$. Meanwhile, $Q$ and $V$ are characterized by

$$\begin{bmatrix} vec(Q) \\ vec(V) \end{bmatrix} = -W^{-1} \begin{bmatrix} vec(\mathbb{D}) \\ vec(\mathbb{L}\mathbb{N} + \mathbb{M}) \end{bmatrix}. \tag{D.46}$$

$W$ is a matrix that contains $I_I$ which is an identify matrix for the shocks

$$W = \begin{bmatrix} I_I \otimes \mathbb{A} & I_I \otimes \mathbb{C} \\ \mathbb{N}' \otimes \mathbb{F} + I_I \otimes (\mathbb{F}P + \mathbb{J}U + \mathbb{G}) & \mathbb{N}' \otimes \mathbb{J} + I_I \otimes \mathbb{K} \end{bmatrix}, \tag{D.47}$$

$\otimes$ stands for tensor product and *vec* for column-wise vectorization.

Once we characterize the law of motion for jump and state variables (Equation D.42), we can follow non-expectational equations to express static control variables as functions of jump and state variables, in order to predict their responses to the shocks in the economy. Static control variables include both real and financial variables. For example, asset dividends and returns are derived from Equations D.21 and D.22. The responses of asset returns $R$ and priced-adjusted consumption $C^p$ to the shocks will determine $D_1, D_2, R_1, R_2$ in Equations D.5, D.6 and D.11 to solve the portfolio choice problem.