

Spatial Consumption Risk Sharing

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Abstract

This paper examines how bilateral economic linkages shape consumption synchronization across economies. Using state-level data from the US, we find that the degree of bilateral consumption risk sharing decreases with geographic distance. To explain this novel fact, we develop an open economy DSGE model that incorporates trade, migration, and finance as channels of risk sharing which are subject to frictions that covary with distance. Calibrated to the US data, the model not only enables us to quantify the magnitude of the frictions in each channel, but also allows us to examine the interaction of the channels and disentangle their effects on the level, volatility, and comovement of consumption. Counterfactual analyses based on the model provide guidance for the design of macroeconomic policies that aim to reduce consumption disparity.

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1 Introduction

Consumption risk sharing allows agents to yield welfare gains by reducing consumption fluctuations caused by idiosyncratic income shocks. However, frictions in economic exchanges impede consumption from being smoothed across space and time. This paper explores the patterns and determinants of risk sharing by exploiting the variation in bilateral economic linkages shaped by geography.

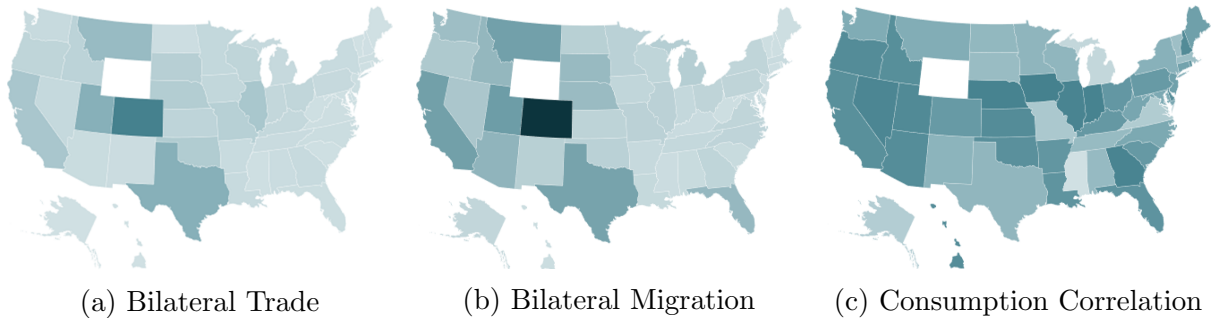
What drives imperfect consumption correlations across economies remains a central question of interest as the phenomenon attests to the failure of complete markets. [Obstfeld and Rogoff \(2000\)](#) consider the low cross-country consumption comovement as a major puzzle in international macroeconomics. Besides trade costs in the commodity market discussed by these authors, migration costs in the labor market and financial frictions in the asset market affect risk sharing since they pose barriers for resources to be freely mobile in the presence of local shocks. While most existing literature studies one channel, this paper extends the workhorse open economy real business cycle model developed by [Backus et al. \(1992\)](#) (BKK) into a unified framework with trade, migration, and finance. This comprehensive framework enables us to examine the interaction of these channels as they jointly influence consumption.

We add a geographic dimension to macro analysis, since bilateral linkages in these channels covary with geographic distance as documented by the gravity model of trade, finance, and migration.¹ Since these channels are important drivers for synchronization, bilateral consumption comovement is also expected to exhibit similar geographic characteristics. To exemplify such patterns, we plot the bilateral economic ties between Wyoming and other states in [figure 1](#) and confirm that ties are generally stronger for neighboring states.² To capture such spatial features, we embed bilateral linkages through channels of risk sharing in a multi-economy DSGE framework that enables us to examine the aggregate influences of different channels in general equilibrium. This RBC framework also contributes to quantitative spatial models surveyed by [Redding and Rossi-Hansberg \(2017\)](#) by evaluating the second moments (variance and covariance) and first moments (level) of macroeconomic fundamentals, both of which are essential for welfare analysis.

¹For example, [Anderson and Van Wincoop \(2003\)](#) develop a theory-grounded econometric framework to revive the gravity model of trade flows across countries. [Portes and Rey \(2005\)](#) document that bilateral equity flows decrease with distance between country pairs. [Lewer and Van den Berg \(2008\)](#) develop and test a gravity model of immigration among OECD countries.

²Detailed data description can be found in [Appendix B](#). Cross-state trade data are sourced from the CFS, migration data are from the IRS, and consumption data are from the BEA. Comprehensive data for state-to-state financial flows are not available to our knowledge.

Figure 1: Wyoming's Bilateral Linkages with Other States



This figure plots bilateral ties between Wyoming (in white) and other states in the U.S. averaged over the period of 1997-2017. A darker color suggests a greater value of bidirectional flows (sum of inflows and outflows) for trade and migration as well as a higher correlation coefficient of real consumption per capita.

This paper focuses on the US state-level analysis, but the general framework can be tailored to other contexts of interest.³ The empirical section consists of two parts. The first establishes a gravity model of consumption risk sharing using output and consumption data from 1977 to 2019. We measure a state's consumption risk sharing as the response of its relative consumption growth to its relative output growth following the macro literature including [Asdrubali et al. \(1996\)](#). Specifically, we compute bilateral risk sharing for all the state pairs and find it is weaker for pairs that are more geographically distant: Every 1% increase in distance deteriorates consumption risk sharing by 0.151 (or 0.402 standard deviations). This spatial characteristic of consumption synchronization points to the existence of barriers to risk sharing influenced by geography. The second empirical analysis examines the 2006 North Dakota (ND) oil boom as an event study to verify the importance of geography in spreading consumption gains. Through panel regressions, we find that bilateral linkages of ND with other states exhibit strong geographic patterns after ND's output boost: ND witnessed greater migration and trade inflows from states located in closer proximity. These states also experienced stronger consumption comovement with ND following the oil shock.

Motivated by the empirical findings, we develop a DSGE model to examine the drivers for this geographic pattern of consumption synchronization. Our model is populated by representative households who reside in different states connected by three channels. In the trade channel, we follow the classic [Armington \(1969\)](#) model to assume that states exchange intermediate goods subject to iceberg trade costs. In the migration channel, we

³For example, the model can be applied to intranational analysis of another country, or international analysis of the European Union which exhibits a high degree of integration for goods, financial, and labor markets. Given that frictions are relatively low across states in the US, our estimates provide a lower bound on the importance of frictions for consumption.

adopt [Artuc et al. \(2010\)](#)'s analysis with modifications by assuming that households make forward-looking migration decisions in response to consumption differentials across states. In the financial channel, we set up a portfolio choice problem and endogenize agents' preference among assets issued by different states. To capture asset market incompleteness, we introduce bilateral financial frictions as transaction costs on asset returns.⁴ When deriving portfolios under frictions, we employ the solution technique developed by [Devereux and Sutherland \(2011\)](#) and [Tille and Van Wincoop \(2010\)](#). The portfolio choice will in turn affect consumption correlations, which allows us to quantify both the magnitude of bilateral financial frictions and the distortion of consumption caused by them.

To illustrate the mechanism of how the three channels of risk sharing jointly shape consumption synchronization, we start with a symmetric two-economy analysis à la BKK. By conducting comparative static analyses, we find that the interaction of the channels yields non-monotonic predictions for the impacts of various frictions on consumption correlations. For example, higher financial frictions, by tilting portfolios towards domestic assets and lowering the reliance of consumption on foreign output, reduce bilateral consumption correlations in general, consistent with the neoclassical model of cross-economy risk sharing ([Lucas \(1982\)](#)). Nevertheless, when financial frictions are so high as to encourage saving that crowds out consumption, population moves out of the state which has experienced a positive productivity shock. These migration outflows equalize consumption per capita across states and hence generate a stronger consumption comovement. This analysis underscores the importance of considering multiple channels of risk sharing in an integrated general equilibrium setting.

To conduct policy analysis with the model, we extend the bilateral to a trilateral framework where we consider the rest of the economy (ROE) which exerts 'multilateral resistance' on a state-pair in the spirit of [Anderson and Van Wincoop \(2003\)](#). To calibrate frictions in the three channels, we use trade and migration shares as well as coefficients of risk sharing as targeted moments. We conduct the estimation for all the state pairs and confirm the geographic feature of bilateral frictions: For a 1% increase in distance, bilateral trade, migration, and financial frictions increase by 0.53%, 0.10%, and 0.23% respectively. Furthermore, we quantify the impacts of frictions on consumption

⁴This modeling assumption follows [Heathcote and Perri \(2013\)](#) and [Tille and Van Wincoop \(2010\)](#), but financial frictions can take alternative forms to asset transaction costs. For example, [Okawa and Van Wincoop \(2012\)](#) discuss the comparability of information frictions and transaction costs in predicting the gravity model of financial flows. Even within a country, there exist such financial frictions that vary at the bilateral level. Empirical evidence for this includes the 'home bias at home' phenomenon documented by [Coval and Moskowitz \(1999\)](#).

through counterfactual analyses. Eliminating three types of bilateral frictions leads to lower consumption volatility, with a reduction of 0.7%, 1.0%, and 0.3% averaged across states when bilateral trade, migration, and financial frictions are turned off respectively. This result supports the argument that reducing barriers to risk sharing yields welfare gains by smoothing consumption fluctuations. These counterfactual analyses also provide guidance for fiscal policies which, by mitigating the impacts of the frictions, reduce consumption inequality. Using an example that studies the direction and magnitude of transfers across states to alleviate the effects of trade costs on the level of consumption, we show that our framework offers a useful tool for the design of macro policies which aim to narrow consumption disparity across space and time.

This paper contributes to the macroeconomics literature on consumption risk sharing by exploiting the bilateral variation across economies influenced by geography. To explain the failure of cross-country risk sharing, international macro literature has examined frictions in the financial channel (e.g. [Baxter and Crucini \(1995\)](#), [Kollmann \(1995\)](#), and [Lewis \(1996\)](#)) or the trade channel (e.g. [Dumas and Uppal \(2001\)](#), [Corsetti et al. \(2008\)](#), and [Eaton et al. \(2016\)](#)). Nevertheless, many works focus on one channel in a two-country framework, which is not ideal to fully characterize the general equilibrium effects. Therefore, this paper is closer to [House et al. \(2018\)](#), [Fitzgerald \(2012\)](#), and [Caliendo et al. \(2018\)](#), who consider multiple channels in a multi-region framework. Compared to these papers, our portfolio choice framework makes it possible to quantify financial frictions at the pair level for cross-sectional comparison and counterfactual analysis. These bilateral financial frictions are important for the spatial pattern of consumption comovement.

In the domestic context, [Asdrubali et al. \(1996\)](#), [Hess and Shin \(1998\)](#), [Crucini \(1999\)](#), [Athanasoulis and Van Wincoop \(2001\)](#), [Del Negro \(2002\)](#), and [Kalemli-Ozcan et al. \(2010\)](#) pioneered the work on risk sharing using the US state-level data. At the micro level, seminal papers including [Storesletten et al. \(2004\)](#) and [Heathcote et al. \(2014\)](#) explore heterogeneous impacts of income on consumption across households. Neither these macro nor micro perspectives focus on the influences bilateral frictions across states on households' consumption and migration decisions. Therefore, our paper complements this literature by considering additional channels of intranational risk sharing.

Lastly, this paper contributes to empirical gravity models. Since being introduced by [Isard \(1954\)](#) and [Tinbergen \(1962\)](#), the model has emerged as a classic framework in the trade literature. In addition to trade, the gravity model has been applied to a wide range of topics including financial assets (e.g. [Portes and Rey \(2005\)](#), [Martin and Rey \(2004\)](#), [Aviat and Coeurdacier \(2007\)](#), and [Okawa and Van Wincoop \(2012\)](#)) and

population flows (e.g. [Lewer and Van den Berg \(2008\)](#) and [Ramos and Suriñach \(2017\)](#)). Nevertheless, less is known about the effects of distance on macroeconomic fundamentals. Our paper, together with [Chertman et al. \(2020\)](#) for cross-country analysis, adds to this literature by exploring the role of geographic distance in shaping consumption patterns.

The remainder of the paper proceeds as follows: Section 2 empirically explores the influence of geographic distance on consumption comovement. Section 3 develops a theoretical framework to examine the magnitude and impact of frictions that covary with geography in the channels of consumption risk sharing. Section 4 concludes.

2 Empirical Motivation

This section empirically establishes the importance of geography for consumption risk sharing. First, we use the US state-level consumption and output data to compute the degree of bilateral risk sharing and explore its sources of variation including distance. Second, we conduct an event study of the 2006 North Dakota oil discovery to verify the role of geography in spreading consumption gains from a local shock.

We measure consumption risk sharing as the response of an economy’s relative consumption growth to its relative output growth following the macro literature such as [Asdrubali et al. \(1996\)](#) and [Kose et al. \(2009\)](#). In particular, we focus on bilateral risk sharing so that we can exploit pair-specific factors including geographic distance in order to examine the patterns and determinants of consumption comovement across economies. Specifically, we evaluate risk sharing between state i and j from

$$\Delta \log c_{it} - \Delta \log c_{jt} = \alpha_{ij} + \beta_{ij}(\Delta \log y_{it} - \Delta \log y_{jt}) + \epsilon_{ijt}, \quad (1)$$

where $\Delta \log c_{it}$ ($\Delta \log c_{jt}$) and $\Delta \log y_{it}$ ($\Delta \log y_{jt}$) denote the growth of log real per-capita consumption and output of state $i(j)$ at time t . The coefficient β_{ij} measures the degree of bilateral consumption risk sharing. In the case with perfect risk sharing, consumption is equalized regardless of relative output growth, which yields a coefficient of 0. In the opposite case with complete autarky, a state’s consumption is solely determined by its own output, which implies a coefficient of 1. Therefore, a lower β_{ij} suggests a higher degree of bilateral risk sharing.

The data using which we evaluate equation 1 are obtained from the following sources (see Appendix B for details). The US Bureau of Economic Analysis (BEA) reports real gross state product (GSP) since 1977 and state-level consumption but only since 1997,

which is not ideal for our analysis of risk sharing that requires long-horizon data. Therefore, we follow [Asdrubali et al. \(1996\)](#)'s method of constructing state-level consumption by rescaling state-level retail sales by the country-level ratio of private consumption to retail sales, both of which are available from the BEA. Moreover, we use [Nakamura and Steinsson \(2014\)](#)'s state-level inflation series to convert nominal to real consumption.

Panel A of table 1 presents the summary statistics of bilateral correlations of HP-filtered real consumption and output per capita (in logs). The mean bilateral output correlation is 0.422 which is higher than the consumption correlation 0.340. This stylized fact across states is consistent with that across countries, which is listed as a puzzle in international macroeconomics ([Obstfeld and Rogoff \(2000\)](#)) since the empirical fact contradicts the theoretical prediction in complete markets. This paper uses domestic data to understand the drivers for consumption synchronization, which also potentially sheds light on the puzzle in the international context.

We establish an empirical gravity model of risk sharing by deriving a cross-sectional prediction for consumption comovement across states. In particular, we explore the implications of geographic distance for bilateral consumption risk sharing by conducting a two-stage regression. In the first stage, we follow equation 1 to estimate the bilateral risk-sharing coefficients for all the state pairs over the sample period. Panel B of table 1 summarizes the statistics of the estimated coefficients $\hat{\beta}_{ij}$. The mean and median values are 0.515 and 0.501 respectively. The fact that $\hat{\beta}_{ij}$ is between 0 and 1 implies imperfect cross-state consumption risk sharing. In the second stage, we regress the estimated $\hat{\beta}_{ij}$ on the log of geographic distance:

$$\hat{\beta}_{ij} = \alpha + \gamma \log(dist_{ij}) + \Gamma X_{ij} + \nu_{ij}. \quad (2)$$

Our hypothesis is that state pairs with greater geographic distance exhibit weaker consumption risk sharing, since bilateral economic exchanges which facilitate consumption comovement potentially face frictions that increase with bilateral distance. γ in equation 2 is therefore expected to be positive.

To test the hypothesis with regression 2, we compile the following variables. We measure cross-state geographic distance by applying the Haversine formula to state capitals' longitude and latitude. In addition, we consider the distance based on the Commodity Flow Survey (CFS) to verify the robustness of our empirical findings.⁵ The results

⁵The CFS reports the shipment mileage between origin and destination ZIP code points for commodity flows used for domestic expenditure within the US. We use the average mileage of shipments between two states to calculate this CFS-based bilateral distance. See table A.2 for this robustness check.

reported in table 2 confirm our hypothesis that bilateral geographic distance and risk-sharing coefficients are significantly and positively correlated. In column (1), when distance rises by 1%, bilateral risk sharing weakens by 0.151 (or 0.402 standard deviations). In column (2) we control for state pairs' time-averaged GSP per capita and find that risk sharing is stronger for states with higher income levels. Therefore, bilateral risk sharing covaries with distance and income per capita in the same direction as in the classic gravity model of international trade. In column (3) we consider other geographic variables of the state pair including the product of their land sizes in square miles (in logs), the number of mainland and coastal states, a contiguity dummy which equals one for state pairs sharing borders, and the total number of neighboring states to capture the state pair's multilateral ties with adjacent states.⁶ Besides, we have the total number of Metropolitan Statistical Area (MSA) and the number of MSA that geographically spans the state pair. MSA matters for the percentage of commuters whose location of residence and consumption differs from location of income.

Furthermore, we consider political and industrial proximity as potential factors for risk sharing based on the macro literature.⁷ We measure a state's position on the political spectrum based on whether its voters chose a Republican or a Democratic candidate ($Pol_{it} = 0$ or 1) during presidential elections from 1976 to 2020, and take a state-pair's squared difference in the time-averaged values (\bar{Pol}_i) to measure political remoteness

$$Pol_{ij} = (\bar{Pol}_i - \bar{Pol}_j)^2. \quad (3)$$

For the dissimilarity of industrial profiles, we compute a state-pair's sectoral composition of output and aggregate the squared difference over sectors

$$Ind_{ij} = \sum_{s=1}^S (b_{i,s} - b_{j,s})^2, \quad \text{where} \quad b_{i,s} = \frac{\bar{Y}_{i,s}}{\sum_{s=1}^S \bar{Y}_{i,s}}. \quad (4)$$

$\bar{Y}_{i,s}$ denotes the output of sector s in state i averaged over the sample period sourced

⁶The number of mainland and coastal states takes values 0, 1, or 2 for a pair of states. Mainland states refer to the 48 contiguous states. Coastal states refer to the states that are not landlocked and instead have a coastline.

⁷For example, Parsley and Popper (2021) document stark business cycle asynchronicity among blue versus red states in the US, and reason that differences in fiscal policies potentially explain how political division shapes this pattern of risk sharing. Meanwhile, the complementarity of industrial structures influences and is influenced by economies' output and consumption synchronization, according to the empirical findings of Kalemli-Ozcan et al. (2003).

from the BEA.⁸ As suggested by table 2 column (4), state pairs with greater political similarity and industrial dissimilarity exhibit a higher level of risk sharing, consistent with the results documented by Parsley and Popper (2021) and Kalemli-Ozcan et al. (2003). Meanwhile, the coefficient of distance remains economically and statistically significant.

Table 1: Summary Statistics of Output, Consumption, and Risk Sharing Coefficients

	Mean	Median	Std. Dev.	Observations
A. Bilateral Correlation				
Output	0.422	0.479	0.316	1225
Consumption	0.340	0.388	0.329	1225
B. Risk Sharing Coefficient				
$\hat{\beta}_{ij}$	0.515	0.501	0.292	1225

Bilateral correlation of output (consumption) is calculated as the correlation of HP-filtered real output (consumption) per capita in logarithms across all the state pairs over the sample period from 1977-2019. $\hat{\beta}_{ij}$ is estimated as the response of the relative consumption growth to the relative output growth as specified in equation 1.

In addition to the baseline estimation described above, we perform two sets of tests to verify the robustness of the gravity model. First, we consider alternative data sources for state-level consumption, price, and bilateral geographic distance. Second, we reconstruct measures of bilateral risk sharing after controlling for 1) state-level demographic variables which potentially shift aggregate demand over time including age, gender ratio, and education level, and 2) states' distinct exposure to aggregate country-level shocks. The results reported in Appendix A suggest that our finding remains robust.

The gravity model of risk sharing established above suggests the existence of frictions in the channels of risk sharing that covary with distance. We test for the underlying mechanism by examining the joint influences of distance and potential channels including trade, migration, and finance on consumption. Specifically, we compute bilateral linkages in these channels as the state-pair's mean value of bidirectional flows averaged over time. For example, bilateral trade linkages (Z_{ij}) are calculated with trade flows in logarithm

$$Z_{ij} = \sum_{t=1}^T \frac{\log(trd_{ijt}) + \log(trd_{jit})}{2T}. \quad (5)$$

Bilateral trade and migration flows are obtained from the CFS and IRS respectively (see

⁸To calculate sectoral shares in state-level output ($b_{i,s}$), we use the real sectoral output series (SAGDP9N) from the BEA, which reports data based on the 2012 North American Industry Classification System (NAICS) at the 3-digit level.

Table 2: Spatial Pattern of Risk Sharing

Dep. Var: $\hat{\beta}_{ij}$	(1)	(2)	(3)	(4)
$\log(d_{ij})$	0.151 *** (0.010)	0.156 *** (0.010)	0.220 *** (0.012)	0.211 *** (0.012)
$\log(\bar{y}_i \cdot \bar{y}_j)$		-0.099 *** (0.032)	-0.061 * (0.035)	0.052 (0.038)
Land Area			-0.038 *** (0.006)	-0.022 *** (0.006)
Mainland			0.117 *** (0.025)	0.079 *** (0.024)
Coastal			0.018 (0.014)	0.023 * (0.014)
Contiguity			0.128 *** (0.033)	0.102 *** (0.033)
Number of Neighboring States			-0.002 (0.004)	-0.005 (0.004)
Number of MSA			0.001 (0.001)	-0.002 * (0.001)
Number of Shared MSA			0.021 (0.023)	0.022 (0.022)
Industrial Dissimilarity (Ind_{ij})				-5.480 *** (0.754)
Political Dissimilarity (Pol_{ij})				0.069 ** (0.032)
Observations	1225	1225	1225	1225
R^2	0.161	0.169	0.255	0.288

Robust standard errors in parentheses. *** significant at 1%. The dependent variable is the risk sharing coefficient $\hat{\beta}_{ij}$, which is estimated using the real consumption and output data over 1977-2019. d_{ij} denotes the geographic distance between state i and j . \bar{y}_i denotes the time-averaged output per capita of state i . Other control variables include a state-pair's geographic characteristics as well as political and industrial dissimilarity.

Table 3: Interaction of Distance with Different Channels of RS

Dep. Var: $\hat{\beta}_{ij}$	(1)	(2)	(3)	(4)
Indep. Var Z_{ij}	–	Trade	Migration	Finance
$\log(d_{ij})$	0.211 *** (0.012)	0.423 *** (0.044)	0.429 *** (0.054)	0.218 *** (0.012)
Z_{ij}		0.206 *** (0.042)	0.268 *** (0.053)	5.4e-08 *** (1.8e-08)
$\log(d_{ij}) \times Z_{ij}$		-0.022 *** (0.006)	-0.023 *** (0.008)	-7.4e-09 *** (2.5e-09)
Other Gravity Var	Y	Y	Y	Y
Observations	1225	1225	1225	1225
R^2	0.288	0.307	0.360	0.293

Robust standard errors in parentheses. *** significant at 1%. The dependent variable is the estimated risk sharing coefficient $\hat{\beta}_{ij}$. d_{ij} denotes the geographic distance between state i and j . Z_{ij} is the state-pair’s mean value of bidirectional trade, migration, and financial flows averaged over time. Trade data are from the CFS, migration data are from the IRS, finance data here are based on FDIC’s amount of deposit collected by financial institutions with a branch in one state and headquarter in another. Trade and migration flows are in logarithm and financial flows are in levels to keep the full sample of state pairs (given 700 out of 1225 observations as zeros). Other gravity variables include all the independent variables listed in table 2.

Appendix B for details). Financial flows are based on FDIC’s deposit amount collected by financial institutions headquartered in one state and located in another.⁹ Table 3 reports the regression results with estimated $\hat{\beta}_{ij}$ as the dependent variable and all the gravity variables from table 2 plus the three bilateral linkages as independent variables. The results show that $\hat{\beta}_{ij}$ still increases in distance but decreases in its interaction terms with bilateral trade, migration, and finance. The negative coefficients of the interaction terms suggest that these three channels alleviate the negative impacts of geography on cross-state consumption risk sharing.

After exploring the general covariance between risk sharing and distance using long-term data, we conduct an event study to verify the importance of geography for bilateral economic linkages including consumption comovement. Specifically, we focus on the North Dakota oil supply shock that started from the surprising discovery of oil by a petroleum geologist in 2006. The discovery provides a natural experiment for us to eval-

⁹Comprehensive data for state-to-state financial flows are not existent to our knowledge, but the Federal Deposit Insurance Corporation (FDIC) bank statistics lists branch locations and deposits of its insured financial institutions. States i and j are hereby deemed to exhibit stronger financial ties when banks headquartered in i collect more deposits from branches located in j . It is the among the most comprehensive public data to document financial linkages across states. However, given the geographic concentration of the US banking industry and under-representation of bank deposits in total financial exchanges, it is not sufficient to empirically reflect bilateral financial flows or to structurally estimate the theoretical model with in the next section.

uate the impacts of a local output boost. The rapid oil extraction since the discovery has not only fueled the economic boom of North Dakota (ND hereafter) but also positively affected other states through their economic exchanges with ND.

To establish the spatial feature of economic linkages in the wake of the oil shock, we run a panel regression with all the state pairs formed by ND over the period from 1991 to 2019 where migration and trade data are available. The regression is specified as follows

$$X_{ijt} = \alpha_0 + \alpha_1 Oil_t + \sum_{m=1}^T \alpha_{2m} Oil_{t-m} + \alpha_3 \log(dist_{ij}) + \sum_{n=0}^T \alpha_{4n} Oil_{t-n} \times \log(dist_{ij}) + \alpha_{5t} I_t + \alpha_{6j} I_j + \zeta_{ijt}. \quad (6)$$

X_{ijt} represents bilateral variables of interest including migration flows (mig_{ijt}), trade values (trd_{ijt}), and relative per-capita consumption growth between state i as ND and j as any other state.¹⁰ For migration and trade, we focus on the log of ND's population and goods inflows from other states to capture the spillover of the positive shock. For the relative consumption growth, we consider both $\Delta c_{ijt} \equiv \Delta \log c_{it} - \Delta \log c_{jt}$ and $\Delta \tilde{c}_{ijt} \equiv (\Delta \log c_{it} - \Delta \log c_{jt}) - (\Delta \log y_{it} - \Delta \log y_{jt})$. The latter can be regarded as the consumption growth unexplained by the output growth of ND relative to other states, which provides a more robust measure of consumption risk sharing. To isolate the responses of these variables to the oil shock as deviations from their long-term trend, we take the difference between the realization of these bilateral variables at time t and their mean values over the sample period, and use these demeaned values as dependent variables. In addition, we control for time fixed effects (denoted as I_t) which reflect the aggregate shocks that happen at time t and state fixed effects (I_j) to control for cross-state differences independent of the oil shock. Oil_t is a binary variable which is unity when t represents year 2006 and zero otherwise. We also consider medium-run effects of the shock by including lagged dummies Oil_{t-m} which equal one when the oil shock happens m years ago. In the baseline case, we set the maximum number of lags as three years for migration and consumption, and as eleven years for trade to get sufficient observations under its five-year data frequency. The key variable of interest to verify the importance of geography for economic linkages is $\sum_{n=0}^T \alpha_{4n}$, the linear combination of coefficient estimates for the interaction terms of the oil shock and bilateral distance.

Table 4 reports the regression results. Based on the interaction terms, bilateral economic linkages exhibit strong spatial patterns. As is shown in columns (1) and (2), a 1%

¹⁰We do not include finance in this event analysis due to the lack of financial data. Even with FDIC's banking data, ND's observations are very scarce since it is not a major hub for the banking industry.

Table 4: Bilateral Linkages after the Oil Shock

Dep. Var:	(1)	(2)	(3)	(4)	(5)	(6)
	$\log(trd)$	$\log(mig)$	$\log(trd)$	$\log(mig)$	Δc	$\Delta \tilde{c}$
Oil_t		0.124		0.123	-0.010	0.014
		(0.465)		(0.473)	(0.051)	(0.055)
$\sum_{m=1}^T Oil_{t-m}$	1.883 *	-0.974	1.836 *	-0.974	-0.045	0.098
	(0.967)	(0.599)	(0.992)	(0.608)	(0.079)	(0.064)
$\log(dist)$	0.012	0.013	0.006	0.014	-0.002	-0.001
	(0.075)	(0.014)	(0.352)	(0.057)	(0.008)	(0.009)
$\sum_{n=0}^T Oil_{t-n} \times \log(dist)$	-0.578 *	-0.394 ***	-0.339	-0.393 ***	0.049 ***	0.040 **
	(0.325)	(0.146)	(0.363)	(0.149)	(0.017)	(0.017)
State FE	N	N	Y	Y	Y	Y
Time FE	Y	Y	Y	Y	Y	Y
Observations	244	1,360	244	1,360	1,372	1,372
R^2	0.657	0.645	0.688	0.645	0.650	0.676

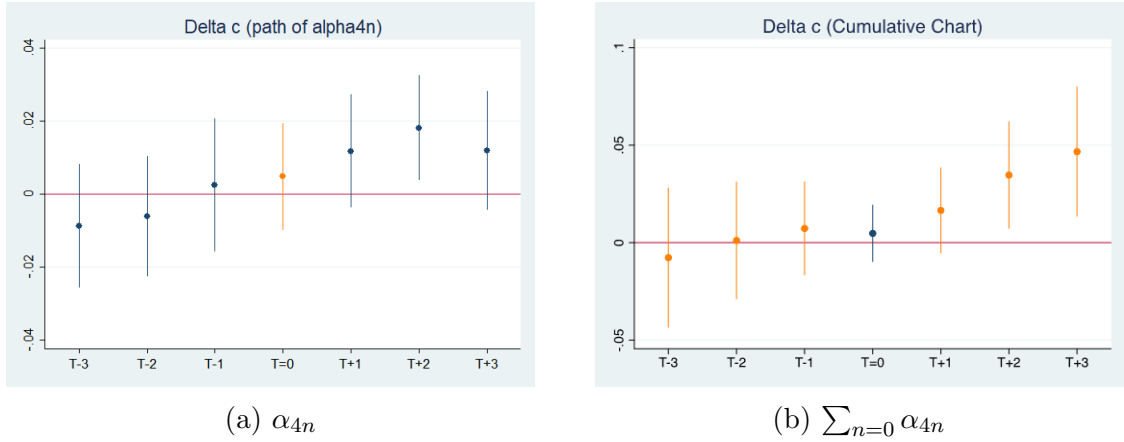
Robust standard errors in parentheses. *** significant at 1%, ** at 5%, and * at 10%. The dependent variables include North Dakota (ND)'s demeaned migration and trade inflows in logs from other states, as well as ND's per-capita consumption growth relative to other states (Δc), and the relative consumption adjusted for output growth ($\Delta \tilde{c}$). $\log(dist)$ denotes the geographic distance between ND and other states. Oil_t is a dummy variable for the oil shock to ND in 2006. Its coefficient is missing in columns (1) (3) since the CFS trade data are not available that year.

increase in bilateral geographic distance lowers trade and migration flows from another state to ND by 0.578% and 0.394% respectively due to the oil shock.¹¹ This finding points to the barriers in these two channels that covary with geography which limit the scope of positive influences brought forth by ND's economic success. Consequently, residents from distant states are constrained from physically moving to or exporting goods to the booming state. Such barriers can also account for the spatial pattern of consumption. As is reported in columns (5) and (6), ND's per-capita consumption growth is larger in magnitude relative to that of more distant states. From column (5), a 1% increase in distance raises ND's relative consumption growth driven by its oil shock by 0.049%. Figure 2 plots the time path of α_{4n} and its cumulative change, which shows a noticeable slope increase after the oil shock. For example, the cumulative consumption growth three years after the shock in Nebraska is 8.7% higher than in Florida. This result, which suggests that ND's consumption is more synchronized with neighboring states', indicates that geography plays an essential role in shaping the variation in consumption comovement. The result remains robust in column (6) where we adjust consumption for output differentials, which further implies that consumption risk sharing deteriorates when distance rises.

To conclude this empirical section, both the gravity model analysis and the ND event

¹¹These results from columns (1) and (2) become weaker in columns (3) and (4) where state fixed effects are added, particularly given the limitation of trade data with low frequency and high sparsity.

Figure 2: Time Series Path of α_{4n} for Relative Consumption Growth



This figure plots the time series pattern of the coefficient estimate α_{4n} when the relative consumption growth Δc is the dependent variable and the interaction term for the oil shock and distance is the independent variable (column (5) in table 4). (a) shows α_{4n} 's estimate and confidence interval at each time point, where $T = 0$ represents year 2006 where the oil shock happened. (b) shows cumulative changes $\sum_{n=0} \alpha_{4n}$ over time.

study verify that geographic distance is important for consumption synchronization. We build a structural model in the next section to explain this spatial pattern of consumption.

3 Theoretical Model

This section develops a model to explain the potential influences of geography on consumption through trade, migration, and financial channels. Section 3.1 describes the model setup. Section 3.2 discusses mechanism of how different channels interact to jointly influence consumption in a symmetric two-state scenario. Section 3.3 provides quantitative analyses to deliver fiscal policy implications in a multi-state setting.

3.1 Setup

The economy is populated by a continuum of infinitely-lived homogeneous households which reside in different states indexed $i \in \{1, 2, \dots, \mathcal{I}\}$. States are interconnected through trade, migration, and finance channels.

Each state produces two intermediate goods: tradables (T) and nontradables (NT). The production of intermediate goods in state i sector $s \in \{T, NT\}$ combines capital

$K_{is,t}$ and labor $L_{is,t}$ with a Cobb-Douglas technology:

$$Y_{is,t} = A_{i,t} K_{is,t}^\alpha L_{is,t}^{1-\alpha}. \quad (7)$$

The state-level productivity $A_{i,t}$ which constitutes a vector $A_t = [A_{1,t}, A_{2,t}, \dots, A_{\mathcal{I},t}]$ follows a joint AR(1) process subject to shocks $\epsilon_t = [\epsilon_{1,t}, \epsilon_{2,t}, \dots, \epsilon_{\mathcal{I},t}]$ with a persistence coefficient matrix ρ and a contemporaneous covariance matrix Σ :

$$A_t = \rho A_{t-1} + \epsilon_t. \quad (8)$$

The final goods for consumption consist of tradables $C_{iT,t}$ and nontradables $C_{iNT,t}$:

$$C_{i,t} = C_{iT,t}^\nu C_{iNT,t}^{1-\nu}, \quad (9)$$

where ν is the weight of tradables. Similarly, the final goods for investment, with price denoted as $P_{Ii,t}$, tradables' weight as ν_I , and quantity $I_{i,t}$ specified as

$$I_{i,t} = I_{iT,t}^{\nu_I} I_{iNT,t}^{1-\nu_I}, \quad (10)$$

add to the capital stock in state i subject to depreciation δ

$$K_{i,t} = (1 - \delta)K_{i,t-1} + I_{i,t}. \quad (11)$$

The market clearing conditions for factors of production and for nontradable goods in state i are respectively given by

$$K_{i,t} = K_{iT,t} + K_{iNT,t}, \quad L_{i,t} = L_{iT,t} + L_{iNT,t}, \quad (12)$$

$$Y_{iNT,t} = C_{iNT,t} + I_{iNT,t}. \quad (13)$$

Meanwhile, tradable goods for consumption and investment will be a CES bundle of intermediate goods sourced from all the states:

$$X_{iT,t} = C_{iT,t} + I_{iT,t}, \quad \text{where} \quad X_{iT,t} = \left(\sum_{j=1}^{\mathcal{I}} X_{ji,t}^{\frac{\phi-1}{\phi}} \right)^{\frac{\phi}{\phi-1}}. \quad (14)$$

However, trade from j to i is subject to an iceberg cost $\tau_{ji} \geq 1$, which together with the

price of j 's output $p_{j,t}$, appears in the aggregate price of tradables in state i :

$$P_{iT,t} = \left[\sum_{j=1}^{\mathcal{I}} (\tau_{ji} p_{j,t})^{1-\phi} \right]^{\frac{1}{1-\phi}}. \quad (15)$$

Based on the price, bilateral trade flows from j to i at t follow

$$X_{ji,t} = \pi_{ji,t} X_{iT,t}, \quad \text{where} \quad \pi_{ji,t} = \left(\frac{\tau_{ji} p_{j,t}}{P_{iT,t}} \right)^{-\phi}. \quad (16)$$

In addition to trade, states are connected through finance. In modeling the asset market, we develop and solve a portfolio choice problem following the asset home bias literature. The main purpose of setting up the portfolio choice problem is to capture the variation of bilateral asset positions in a multi-economy setting.¹² The bilateral variation requires modeling asset holdings and financial frictions at state-pair instead of state-specific levels. Therefore, we introduce bilateral financial friction $e^{-f_{ij}}$ as an iceberg transaction cost when state j repatriates financial returns from state i .¹³ Following Coeurdacier and Rey (2013) and Heathcote and Perri (2013), we assume that each state issues equities, whose dividend payout is capital income net of investment expenditure

$$D_{i,t} = \alpha p_{i,t} Y_{i,t} - P_{Ii,t} I_{i,t}, \quad (17)$$

where $Y_{i,t} = Y_{iT,t} + Y_{iNT,t}$ is the aggregate output in state i . The returns to i 's assets include these dividends and the changes in asset prices denoted as $q_{i,t}$:

$$R_{i,t} = \frac{q_{i,t} + D_{i,t}}{q_{i,t-1}}. \quad (18)$$

We assume there is a mutual fund in every state which makes investment decisions on behalf of its households. The mutual fund constructs a portfolio of assets to maximize the expected lifetime utility from consumption of a household living in the state. In

¹²Empirical evidence for this bilateral variation includes the gravity model of cross-country financial flows (Portes and Rey (2005)), the 'home bias at home' phenomenon in the domestic context (Coval and Moskowitz (1999)), and the FDIC banking statistics including the results from table 3 in this paper.

¹³Modeling transaction costs is not the only way to introduce frictions in the financial channel. In particular, Okawa and Van Wincoop (2012) discuss alternative bilateral financial frictions, including information costs, which can also rationalize the geographic patterns of financial flows.

particular, its objective function is

$$\max \sum_{t=0}^{\infty} \beta^t \frac{c_{i,t}^{1-\sigma}}{1-\sigma}, \quad (19)$$

where $c_{i,t}$ denotes consumption per-capita of state i at time t . A household has the right to an equal share of the fund as long as it resides there.¹⁴ To solve the portfolio choice problem, we use the [Devereux and Sutherland \(2011\)](#) and [Tille and Van Wincoop \(2010\)](#) solution method which combines a second-order approximation of the Euler equations and a first-order approximation of other model equations. Specifically, we evaluate state i 's Euler equation

$$E_t \left[\frac{U'(c_{i,t+1})}{P_{i,t+1}} R_{i,t+1} \right] = E_t \left[\frac{U'(c_{i,t+1})}{P_{i,t+1}} e^{-f_{ji}} R_{j,t+1} \right], \quad \forall j \in [1, \mathcal{I}], \quad (20)$$

and take its difference from state j 's Euler equation to derive a portfolio determination equation (see [Appendix C.2](#) for the derivation in an example with three states):

$$E_t [\sigma(\hat{c}_{i,t+1} - \hat{c}_{j,t+1}) + (\hat{P}_{i,t+1} - \hat{P}_{j,t+1})] \hat{R}_{x,t+1} = \frac{1}{2} \mathcal{F}, \quad (21)$$

where a hat above a variable denotes its log-deviation from the steady state of the economy. $P_{i,t}$ denotes i 's price level, $R_{x,t+1}$ is the vector of excess financial returns, and \mathcal{F} is a matrix of financial frictions. If markets are complete such that the Backus-Smith condition holds:

$$E_t [\sigma(\hat{c}_{i,t+1} - \hat{c}_{j,t+1}) + (\hat{P}_{i,t+1} - \hat{P}_{j,t+1})] = 0, \quad (22)$$

the implied financial frictions in matrix \mathcal{F} should equal zero. Therefore, we infer the magnitude of bilateral financial frictions from [equation 21](#) based on consumption patterns. Since these financial frictions are estimated as the wedge that generates the deviation of consumption from the prediction derived under complete markets, they can be interpreted as all the barriers to financial arrangements that cause market incompleteness impairing consumption risk sharing.¹⁵

¹⁴To simplify the portfolio choice problem, we assume households are myopic and expect themselves to stay in the state when deciding on saving for the next period. Under this assumption, households only care about the expected consumption per-capita in their state of residence during the next period, based on which the local mutual fund makes investment decisions ([19](#)). A future extension of this baseline scenario is to relax the assumption and allow households to consider their own migration probabilities which prompt them to reduce saving and raise current consumption when making investment decisions.

¹⁵This estimation strategy based on consumption data allows us not to take a strong stand on the

If $\alpha_{j,i,t}$ denotes i 's holding of j 's assets derived from the portfolio choice problem, and state \mathcal{I} 's asset is a numeraire asset whose return is $R_{\mathcal{I},t}$, state i 's wealth position follows

$$\mathcal{W}_{i,t+1} = e^{-f_{\mathcal{I}i}} R_{\mathcal{I},t} \mathcal{W}_{i,t} + \sum_j^{\mathcal{I}} \alpha_{j,i,t} (e^{-f_{ji}} R_{j,t} - e^{-f_{\mathcal{I}i}} R_{\mathcal{I},t}) + w_{i,t} L_{i,t} + T_{i,t} - P_{i,t} C_{i,t} - P_{Ii,t} I_{i,t}. \quad (23)$$

$T_{i,t}$ denotes the tax transfer state i receives, which is introduced to capture fiscal policies that also play an essential role in intranational risk sharing.

Households' objective is to maximize their expected lifetime utility. At the beginning of every period, a household living in state i supplies labor, collects labor and financial income, and decides on consumption. It derives utility from consumption $c_{i,t} = \frac{C_{i,t}}{N_{i,t}}$ and disutility from labor hours $l_{i,t} = \frac{L_{i,t}}{N_{i,t}}$ in its state of residence:

$$U_{i,t} = \frac{c_{i,t}^{1-\sigma}}{1-\sigma} - \kappa \frac{l_{i,t}^{1+\eta}}{1+\eta}, \quad (24)$$

where σ captures the degree of risk aversion and $\frac{1}{\eta}$ is the elasticity of labor supply.

After earning and spending its income in state i , the household decides whether and where it wants to migrate. When it makes the decision, it takes into account a non-pecuniary migration cost $d_{ij} \geq 0$ when moving from state i to j . The household collects an idiosyncratic benefit $\omega_j \sim F(\omega)$ from being located in state j at the end of the period. ω_j can be considered as a non-monetary benefit, such as weather and culture, that adds to the utility of living in j . Following [Artuc et al. \(2010\)](#), we assume ω_j is i.i.d across households and drawn from an extreme-value distribution with zero mean:

$$F(\omega) = \exp[-e^{\omega/\theta-\gamma}]. \quad (25)$$

Therefore, a household's expected value of being in state i at time t is

$$V_{i,t} = U_{i,t} + \beta E(V_{i,t+1}) + \sum_j^{\mathcal{I}} \int (\bar{\omega}_{ij,t} + \omega_{jt}) f(\omega_j) \prod_{k \neq j} F(\bar{\omega}_{ik,t} - \bar{\omega}_{ik,t} + \omega_{jt}) d\omega_j. \quad (26)$$

From the three components on the right side of the equation, the expected value consists

exact form these financial frictions take in the real world, which may include borrowing constraints states face when raising funds, informational frictions that prohibit bilateral capital flows, and asset transaction costs that cause market inefficiency. It would be difficult to identify and quantify all of these barriers to financial investment, especially given the lack of comprehensive state-to-state financial data. In a similar spirit, [Fitzgerald \(2012\)](#) also infers asset market frictions from consumption data.

of the current utility the household obtains, the base value of staying in the state, and option value of moving from the state to others in the future. $\bar{\omega}_{ij,t}$ denotes the cutoff benefit that makes the household indifferent between staying in i and moving to j at t :

$$\bar{\omega}_{ij,t} \equiv \beta[E(V_{j,t+1}) - E(V_{i,t+1})] - d_{ij}. \quad (27)$$

Under the distributional assumption of ω , the share of migrants from i to j is

$$m_{ij,t} = \frac{\exp(\bar{\omega}_{ij,t}/\theta)}{\sum_{k=1}^{\mathcal{I}} \exp(\bar{\omega}_{ik,t}/\theta)}, \quad (28)$$

The law of motion for population in state i (denoted as N_i) hence follows

$$N_{i,t} = \sum_{j=1}^{\mathcal{I}} m_{ji,t-1} N_{j,t-1}. \quad (29)$$

To summarize the description of the model, the general equilibrium consists of prices and quantities such that 1) firms set output and price to maximize profit, 2) households choose consumption and migration, mutual funds construct portfolios, to maximize households' expected lifetime utility, and 3) commodity, factor, and asset markets clear.

3.2 Two-state Analysis

This section quantitatively explores the mechanism through which different channels interact with each other and affect consumption risk sharing. We extend the workhorse BKK model by incorporating trade, migration, and financial linkages subject to frictions across two symmetric economies.

In terms of parameterization, the model is calibrated to the US annual data for cross-state analysis. Table 5 summarizes the parametric assumptions under which the baseline two-state framework is solved. Panels (I) and (II) list the parameters whose values are either standard in the macro literature or estimated from the US aggregate economy. For example, we estimate labor share in production $1 - \alpha$ to be 0.59 by dividing the labor earnings by the output data, both from the BEA, over the period of 1977-2019. We set the share of consumption expenditure on tradables (ν) as 0.31 following Johnson (2017), who estimates the value based on the US CPI expenditure data from the BEA. We set the weight of tradables in investment (ν_I) as 0.4 following Bems (2008) based on the OECD input-output table. Moreover, we follow Simonovska and Waugh (2014) and Artuc et al.

(2010) when setting elasticities of trade and migration respectively.

Panel (III) of table 5 characterizes the joint productivity process for a pair of states. We choose Georgia and Ohio (GA and OH for brevity), the median states in terms of output per capita, as our sample of analysis. We first calculate the total factor productivity (TFP) proxied by the Solow residual in each state $i \in \{GA, OH\}$ at time t from

$$\log(A_{i,t}) = \log(Y_{i,t}) - \alpha \log(K_{i,t}) - (1 - \alpha) \log(L_{i,t}), \quad (30)$$

where $Y_{i,t}$ and $L_{i,t}$ are output and number of employees in state i in year t from the BEA. State-level capital stock $K_{i,t}$ is not directly available, so we construct the measure following Garofalo and Yamarik (2002)'s method. Specifically, we apportion national capital stock to states based on their industry-level income data (see Appendix B for details). After we calculate state-level TFP, we detrend the series with the HP filter and estimate a joint AR(1) process, with estimated persistence and covariance matrices of GA and OH's productivity reported in table 5.

Panel (IV) of table 5 lists the values of bilateral frictions calibrated to the state pair. Trade, migration, and financial costs are estimated to match three targeted moments: the mean export-to-output ratio (0.392) and emigrant-to-population ratio (0.028), and the coefficient of risk sharing (0.541) of GA and OH over the sample period. When estimating trade and migration frictions simultaneously, we start with an initial guess for the combination of the two frictions, and solve for the corresponding wage rates and labor hours given the frictions that satisfy the labor market clearing condition. Then we update the guess and repeat the procedure until the model-predicted export-to-output and emigrant-to-population ratios converge to those in the data. In the asset market, we infer financial frictions from consumption based on the Euler equation 21, to capture any barriers in the financial channel that may cause market incompleteness. Calibrating financial frictions with this method involves three steps. First, we obtain the coefficient matrices necessary for portfolio choice from the first-order dynamics of the model (see Appendix C.2 for technical details). Second, we solve for asset holdings under which the model-implied bilateral risk sharing matches that estimated from the data. Third, we use the asset holdings to recover financial frictions from portfolio determination equations.

Given the specified parametrization, table 6 compares the contemporaneous correlations of variables in the calibrated model with those in the data. Panel (I) reports the cross-state comovement of output and consumption. The model performs well in matching empirical moments at both aggregate and per-capita levels. In either case, output

Table 5: Parametrization

Parameter	Description	Value	Source
		(I)	
β	Annual discount factor	0.95	
σ	Coefficient of relative risk aversion	1	Macro
δ	Capital depreciation	0.06	Literature
η	Inverse of elasticity of labor supply	0.5	
		(II)	
ν	Weight of tradables in consumption	0.31	Johnson (2017)
ν_I	Weight of tradables in investment	0.40	Bems (2008)
α	Capital intensity in production	0.41	BEA
$\theta-1$	Elasticity of trade	4.1	Simonovska and Waugh (2014)
ϕ	Elasticity of migration	4.5	Artuc et al. (2010)
		(III)	
ρ	Persistence matrix of productivity	$\begin{bmatrix} 0.65 & 0.06 \\ 0.04 & 0.53 \end{bmatrix}$	Estimated from GA and OH's TFP
Σ	Covariance matrix of shocks	$\begin{bmatrix} 1.21 & 1.25 \\ 1.25 & 2.56 \end{bmatrix} e^{-4}$	
		(IV)	
τ	Trade cost	1.031	Calibrated to match GA and OH's mean export-to-output, emigrant-to-population, and consumption comovement
d	Migration cost	19.58	
f	Financial cost	3e-5	

Table 6: Contemporaneous Correlations of Variables

	Model	Data
	(I) Cross-state Correlation	
Output $\rho(Y_1, Y_2)$	0.85	0.84
Consumption $\rho(C_1, C_2)$	0.79	0.78
Output per capita $\rho(y_1, y_2)$	0.84	0.88
Consumption per capita $\rho(c_1, c_2)$	0.82	0.82
	(II) Correlation with Self Output	
Consumption per capita $\rho(c, y)$	0.95	0.91
Net exports $\rho(NX/Y, Y)$	-0.04	-0.03
Population $\rho(N, Y)$	-0.01	-0.02

This table reports the contemporaneous correlations of HP filtered data and those in the calibrated model. Panel (I) reports the cross-state comovement of output and consumption at the aggregate (denoted as Y_i, C_i) and per capita (denoted as y_i, c_i) levels. Panel (II) reports the comovement of a state's scaled net exports (NX/Y) and population (N) with its own output, as well as the correlation between its consumption and output per capita.

exhibits stronger cross-state synchronization than consumption, consistent with empirical facts. Panel (II) presents the correlation between a state’s own variables with its output per capita. Consumption per capita is highly procyclical while scaled net export (NX/Y) is countercyclical.¹⁶ In addition, the contemporaneous correlation between population and output is negative in both the model and data. Nevertheless, this correlation does not reflect the cumulative effects caused by delayed migration decisions under frictions. To overcome such limitations, we examine the dynamics of variables by plotting impulse response functions (IRFs).

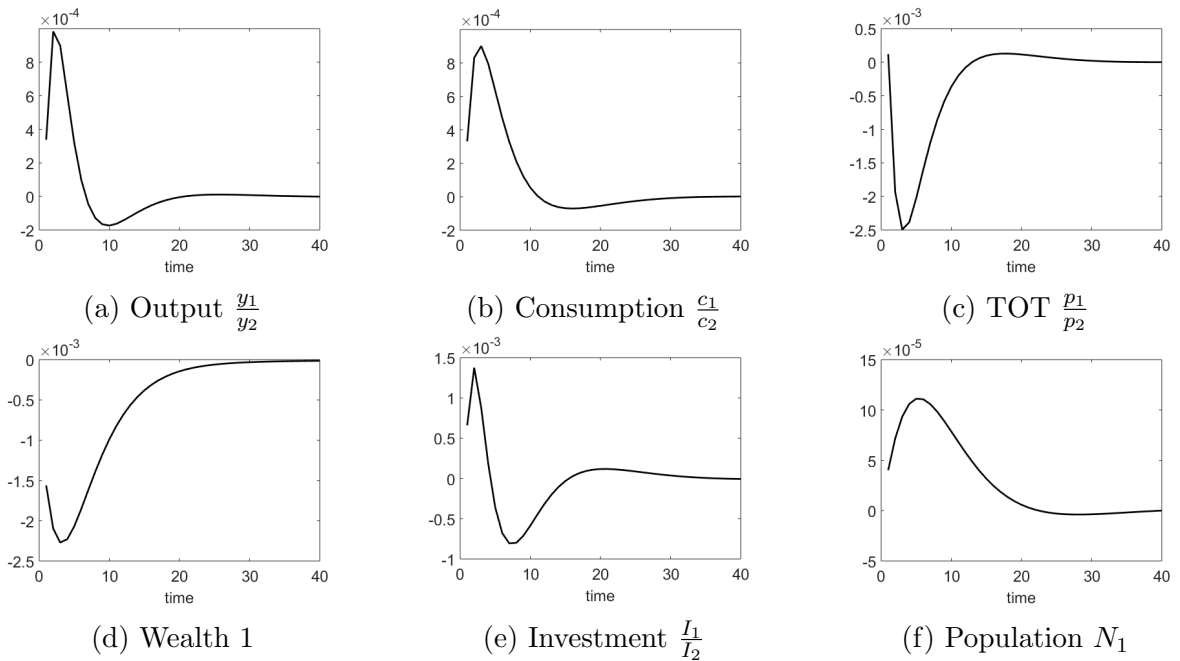
Figure 3 shows the IRFs following a one-standard-deviation innovation to state 1’s productivity. State 1 experiences a stronger output boost in response to its local productivity shock than state 2, as shown in the spike of relative output per capita ($\frac{y_1}{y_2}$) in figure 3a. In comparison, the response of relative consumption per capita ($\frac{c_1}{c_2}$) in figure 3b is not as volatile, which provides evidence for consumption risk sharing through the following channels. In the trade channel (3c), state 1 witnesses a terms-of-trade (TOT) depreciation as its exports become relatively cheaper under increased supply to clear the goods market. This depreciation helps increase the consumption of state 2 by raising its relative nominal income and making its imports more affordable. Meanwhile, state 1 has a negative external wealth position (3d) which suggests that it borrows from state 2. This could be understood from the fact that capital resources are allocated to the more productive economy where returns to capital are higher, which contributes to state 1’s relative investment spike shown in figure 3e. This cross-economy investment financing facilitates risk sharing, as is argued by Heathcote and Perri (2013). Lastly, population flows into state 1 (3f), which raises the number of households among which the increased aggregate consumption is shared and hence helps to equalize consumption per capita across states.

We conduct comparative static analyses by varying frictions in different channels to see how they interact to influence consumption. Figure 4 plots the IRFs when trade cost is 1 (t_{low}) and 2 (t_{high}) times the calibrated value while other parameters remain unchanged. Under a higher trade cost, state 1’s TOT depreciation in 4a is diminished. Turning off this price adjustment in the trade channel limits the consumption gain of state 2, which is reflected in 4b where relative consumption of state 1 becomes more volatile. For bilateral economic exchanges, a higher trade cost not only poses barriers for commodity to move across states in 4c, but also pushes more population to migrate from state 2 to

¹⁶These findings are also consistent with the international stylized facts documented by Mendoza (1991) and Backus et al. (1992).

1 in 4d due to the worsening consumption inequality caused by the trade friction. In this process, households switch from trade to migration as means of consumption risk sharing. Yet, this is not sufficient to leave consumption unaffected as figure 4e suggests that a higher trade cost reduces consumption correlation across states.¹⁷ Besides, the steady-state level of consumption in 4f decreases in trade costs that cause loss of tradable goods during transportation. Based on these results, eliminating trade costs will both raise consumption and facilitate cross-state risk sharing.

Figure 3: Impulse Response Functions after State 1's Positive Productivity Shock

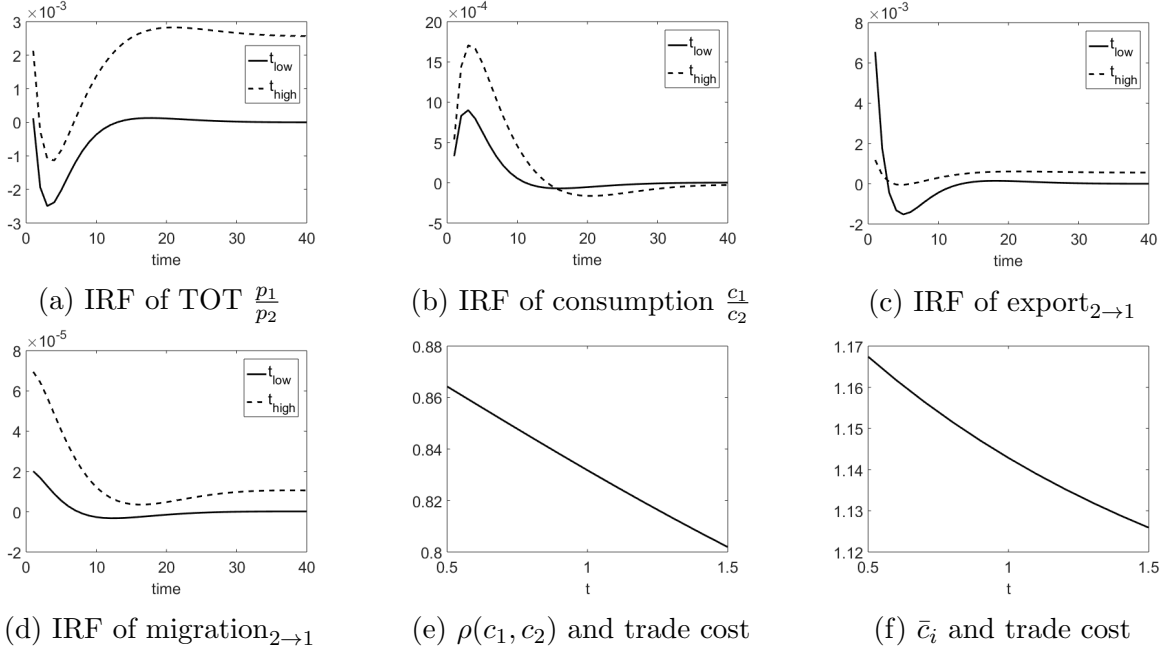


Note: This figure plots the impulse response functions to a one-standard-deviation innovation in state 1's productivity. Variables under examination include the cross-state ratio of output per capita (3a), consumption per capita (3b), price of output or terms-of-trade (3c), and investment expenditure (3e), as well as state 1's external wealth (3d) and population (3f).

We proceed to conduct analysis in the migration channel. Figure 5a suggests a non-monotonic pattern between consumption correlation and migration cost. To understand this pattern, we plot the IRFs when migration cost is 1 (d_{low}), 1.5 (d_{mid}), and 2 (d_{high})

¹⁷We calculate the model-predicted consumption correlation under counterfactual frictions by following three steps. Step 1, we calculate the equilibrium values of all the variables on the real side of the economy under specific trade and migration frictions. Step 2, we solve the portfolio choice problem under financial frictions by evaluating the first-order dynamics of the real-side of the economy and then the second-order expansion of the portfolio equation (see appendix C.2 for details). Step 3, we simulate productivity shocks to the economy that encompasses both real and financial allocations and compute the resulting bilateral consumption comovement.

Figure 4: IRFs and Consumption Moments under Different Trade Costs



Note: Figures 4a-4d plot the impulse response functions to a one-standard-deviation innovation in state 1's productivity. Variables include state 1's terms of trade TOT (4a) and consumption ratio to state 2's (4b), state 2's export (4c) and migration (4d) to state 1. Solid lines are IRFs under calibrated trade cost (t_{low}), while dashed lines are IRFs under counterfactual trade cost whose value is twice as large as the calibrated value (t_{high}). Figure 4e plots the correlation coefficient across states and figure 4f plots the steady-state value for consumption per capita under different multipliers for the calibrated trade cost.

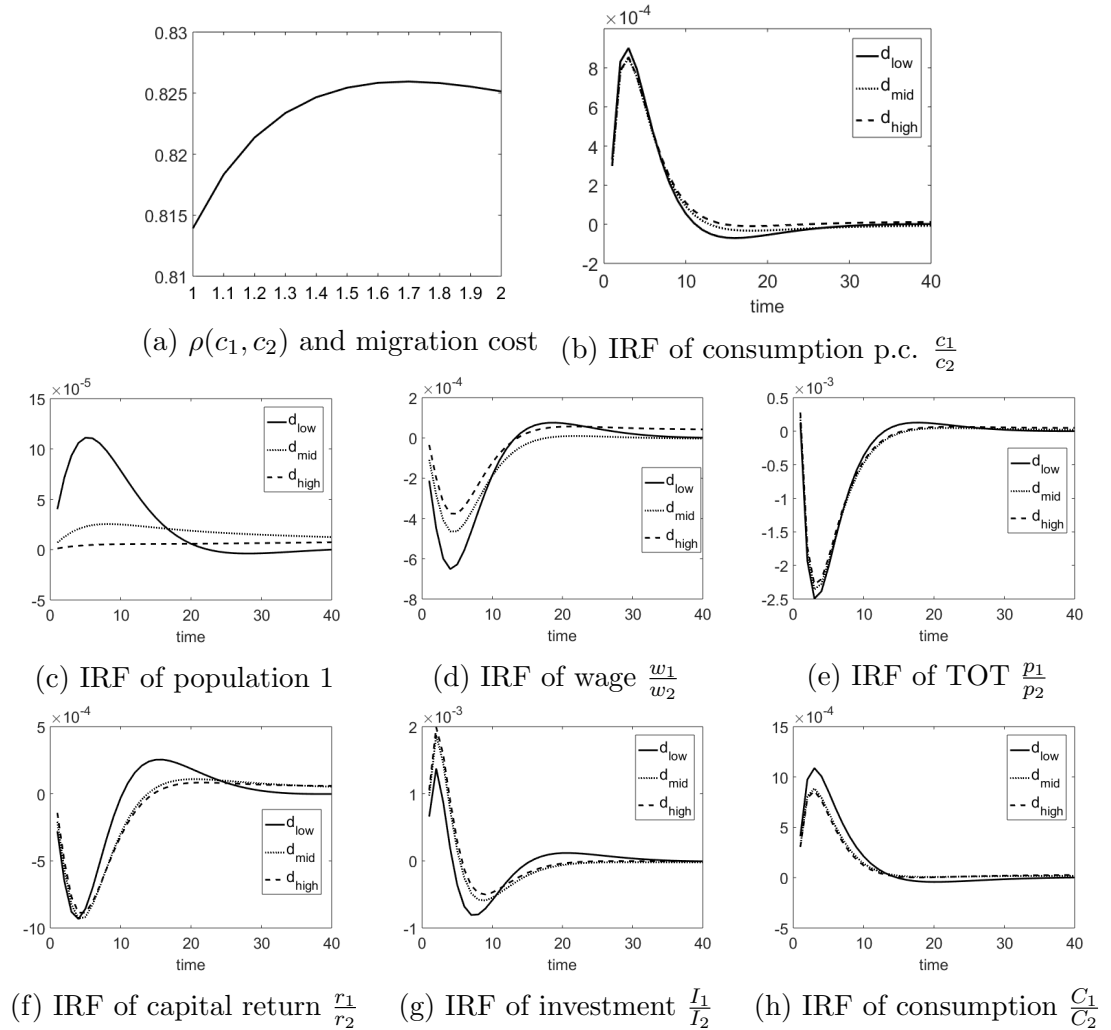
times the calibrated value. When the migration cost decreases from mid to low, more population flows into state 1 after its productivity shock (5e), which causes a larger drop in relative wage (5d). This decline of wage as a production cost exacerbates the TOT depreciation of state 1 (5e), which also reduces state 1's relative nominal marginal product of capital during the initial periods (5f). This lower capital return discourages physical capital investment (5g) and encourages households in state 1 to raise their consumption (5b, 5h) which becomes even larger than state 2's right after the productivity shock. This explains why consumption correlation declines when migration cost decreases if the cost is in a low range. If the migration cost is in a high range, it no longer significantly affects factor prices or consumption-investment decisions (5f-5h). Consumption synchronization is impaired if the migration cost changes from mid to high, because a higher cost deters population from moving, while migration would help narrow the difference in consumption per capita across states. Hence based on the non-monotonic pattern in figure 5a, lowering migration costs will facilitate consumption risk sharing for states faced with high costs,

but not for states that start with low migration costs.

Lastly, we explore the pattern of consumption comovement under different financial frictions. Figure 6a suggests that consumption correlation does not vary monotonically or smoothly with financial frictions. To understand this pattern, we plot the IRFs when the financial friction is 1 (f_{low}), 3 (f_{mid}), and 9 (f_{high}) times the calibrated value. When the financial friction increases from low to mid, consumption comovement becomes weaker. This is because a higher cost of holding foreign assets tilts portfolios more toward domestic assets. Each state's consumption, driven more by its own output performance, is hence less synchronized with each other. Therefore, a higher financial friction strengthens the relative consumption growth of state 1 after its productivity boost (6b), which attracts more population inflows (6c). What causes the discontinuity in figure 6a is the drastic change in the migration pattern when financial friction is even higher. When the friction further increases from mid to high, state 1 has to start saving for its own expenditure, shown as a positive wealth position in (6d). This saving raises investment (6e) but crowds out consumption (6f). Lower aggregate consumption induces population to move out of state 1 (6c), which equalizes consumption per capita across states and generates a higher consumption comovement in figure 6a. In this process, migration replaces finance as a major channel for consumption synchronization when the latter faces greater barriers.

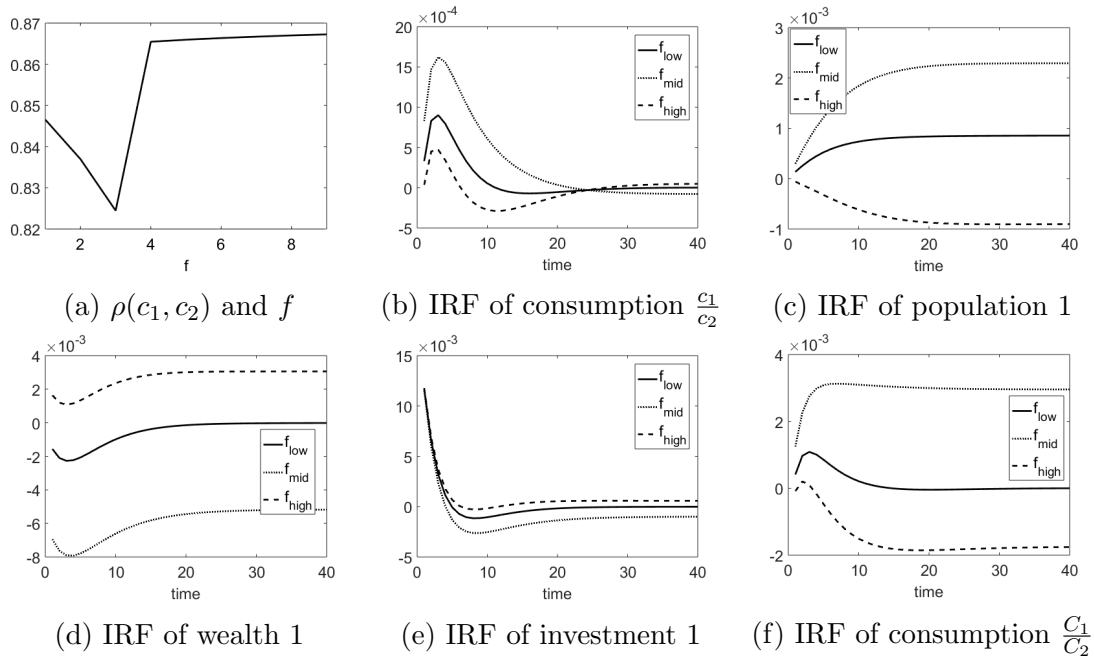
From these comparative static analyses, different channels of risk sharing interact to jointly shape consumption comovement. Examining them in isolation without considering their interplay may yield incorrect policy predictions. The next section builds on these mechanisms to design macro policy that addresses consumption disparity.

Figure 5: Consumption Comovement and IRFs under Different Migration Costs



Note: Figure 5a plots the bilateral correlation of consumption per capita under different multipliers for the calibrated migration cost. Figures 5b-5h plot the impulse response functions (IRFs) to a one-standard-deviation innovation in state 1's productivity. Variables include cross-state ratio of consumption per capita (5b), wage (5d), output price (5e), capital return (5f), investment (5g), aggregate consumption (5h), and state 1's population (5c). Solid, dotted, and dashed lines represent IRFs when the migration cost is 1 (d_{low}), 1.5 (d_{mid}), and 2 (d_{high}) times the calibrated value respectively.

Figure 6: Consumption Comovement and IRFs under Different Financial Frictions



Note: Figure 6a plots the bilateral correlation of consumption per capita under different multipliers for the calibrated financial friction. Figures 6b-6f plot the impulse response functions (IRFs) to a one-standard-deviation innovation in state 1's productivity. Variables include state 1's population (6c), wealth (6d), physical investment (6e), and cross-state ratio of aggregate consumption (6f) and consumption per capita (6b). Solid, dotted, and dashed lines represent IRFs when the financial friction is 1 (f_{low}), 3 (f_{mid}), and 9 (f_{high}) times the calibrated value respectively.

3.3 Multi-state Analysis

This section evaluates the quantitative predictions in an asymmetric multi-state setting to deliver policy implications. Compared to the symmetric two-state case in section 3.2, states have different economic sizes and wealth positions calibrated to the data. A state's change in its net wealth position, which equals the difference between its aggregate expenditure and income especially tax transfers, also reflects other means of risk sharing including fiscal federalism beyond the three channels. Meanwhile, the multilateral framework ensures the clearing of goods, labor, and financial markets in aggregate.

Ideally, a household in state i considers $\mathcal{I} = 50$ states when making economic decisions in the three channels. One computational challenge we face when solving the large-scale DSGE model is that the coefficient matrices that cover all the \mathcal{I} states are badly scaled given states' uneven sizes and sparse bilateral linkages. Therefore, using these matrices to derive portfolio choice with higher-order perturbation yields unreliable

numerical predictions.¹⁸ To overcome this challenge, we propose a trilateral framework that consists of a state-pair and the rest of the economy (ROE) that sums up all the states except for the pair under examination. This trilateral framework, which is applied to all the $\frac{1}{2}\frac{\mathcal{I}}{\mathcal{I}-1} = 1225$ state pairs, enables the analysis of both bilateral linkages between the pair and multilateral resistance exerted on the pair from all the other states in the spirit of [Anderson and Van Wincoop \(2003\)](#). Appendix C provides more details of the quantitative model, including the calibration strategy for different frictions in [C.1](#).¹⁹

To provide a first glance of the estimated frictions in the three channels of risk sharing, we use Wyoming as an example by showing the heatmaps of its estimated bilateral frictions with others in figure 7. Each type of bilateral friction is calculated as the geometric mean of outbound and inbound frictions ($x_{WY,i}$, $x_{i,WY}$, $i \in [1, \mathcal{I}]$, $x \in \{\tau, d, f\}$) between Wyoming (in white) and any other state. In general, states located within a smaller radius from Wyoming exhibit lower frictions with the state. For example, the migration cost between Wyoming and a neighboring state Colorado is the lowest, whose value is approximately 1/3 of that between Wyoming and Hawaii. This spatial pattern is consistent with the observation in figure 1 that Wyoming shows stronger economic linkages with states in closer proximity. However, there are exceptions to the pattern. Idaho, another neighboring state of Wyoming, is estimated to inflict relatively high trade cost under its low trade volume with Wyoming unexplained by the size of its expenditure.

To explore the general spatial pattern of the three frictions, we run bivariate regressions with the estimated frictions as the dependent variables and geographic distance as the independent variable for all the $\frac{\mathcal{I}(\mathcal{I}-1)}{2}$ state pairs:

$$\log(\hat{x}_{ij}) = \alpha_x + \gamma_x \log(\text{dist}_{ij}) + \epsilon_{ij}, \quad x \in \{\tau, d, f\}. \quad (31)$$

As reported in table 7, a 1% rise in distance is associated with a 0.525% increase in trade costs, a 0.100% increase in migration costs, and a 0.232% increase in financial frictions.

¹⁸The badly-scaled coefficient matrices both make the Blanchard-Kahn condition hard to be satisfied and generate extreme values for numerical results even when the condition holds. This issue worsens as solving for portfolio choice requires 2nd-order approximations, which are likely to generate explosive paths even when corresponding linear approximations are stable ([Fernández-Villaverde et al. \(2016\)](#)).

¹⁹In particular, financial frictions are estimated from the consumption data to capture barriers that cause the deviation of consumption from the allocation in complete markets. Table A.5 compares these model-predicted frictions with bilateral banking linkages based on the FDIC data, and find states with stronger banking linkages are predicted to face lower frictions. Although this evidence provides some external validity, financial frictions take many other forms beyond the banking sector, including transaction costs, financial liquidity, and informational frictions in different asset markets. Given the scarcity of state-to-state financial data, estimating bilateral financial friction from consumption which reflects market incompleteness offers much theoretical appeal and flexibility.

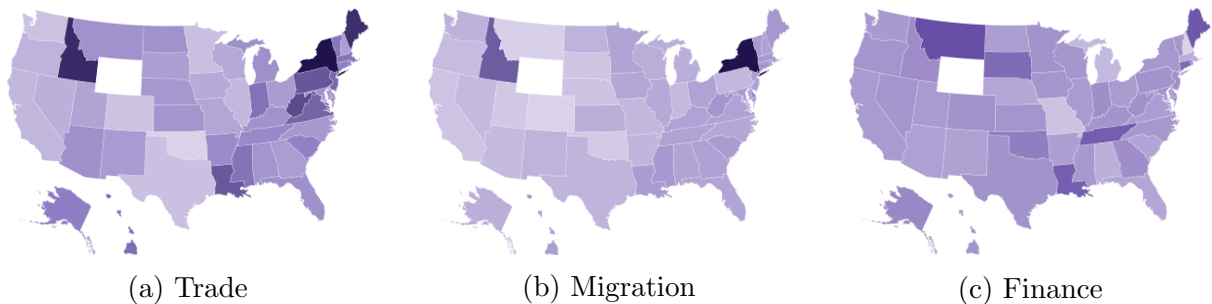
Table 7: Bilateral Frictions and Geographic Distance

Dep. Var: Est. Frictions	$\log(\hat{\tau}_{ij})$	$\log(\hat{d}_{ij})$	$\log(\hat{f}_{ij})$
$\log(dist_{ij})$	0.525 *** (0.047)	0.100 *** (0.01)	0.232 ** 0.097
Observations	2442	2442	2442
R^2	0.041	0.023	0.003

This table reports the regression results of equation 31. Robust standard errors in parentheses, standardized coefficients in brackets. *** significant at 1%, ** significant at 5%. Estimated frictions are missing for few pairs because the eigenvalues computed at the steady state of the model for those pairs do not satisfy the Blanchard and Kahn condition to establish the existence of a unique solution.

These values suggest that trade costs are most sensitive to geography. As the coefficient estimates of distances are all significantly positive, we confirm a key hypothesis of this paper that frictions which impair risk sharing covary with geographic distance between states, which potentially shapes the spatial pattern of consumption synchronization.

Figure 7: Wyoming’s Estimated Frictions with Other States



This figure plots the estimated bilateral frictions between Wyoming (in white) and other states in the U.S. A darker color suggests a higher value of friction. Frictions are calculated as the geometric average of bidirectional frictions (inbound friction to and outbound friction from Wyoming) in each of the channels.

We proceed to quantify the impacts of frictions by conducting counterfactual analyses where we turn off one friction at a time. The median bilateral correlation coefficient of consumption per capita (ρ_c) across state pairs in the sample is 0.401 in the data, and changes to 0.735, 0.395, and 0.429 respectively without bilateral frictions in trade, migration, and finance. The direction of changes is consistent with the two-state analysis in section 3.2 (figures 4-6): while the decrease in trade costs inarguably raises consumption correlation, the reduction in migration or financial frictions yields nonmonotonic predictions. Around the calibrated migration cost for the median state-pair (d_{low} in figure 5), migration exacerbates cross-state consumption inequality following the terms-of-trade adjustment. Therefore, a decline in migration cost leads to a lower consumption corre-

lation in that range of parameter values. In the financial channel, the magnitude of the calibrated friction (f_{low} in figure 6) is not large enough to redirect migration. The financial friction only skews portfolios towards domestic assets and hence reduces the reliance of consumption on foreign economies. For this reason, eliminating financial frictions facilitates cross-state consumption risk sharing.

Turning off these frictions also affects the level and volatility of consumption. Table A.4 reports these median counterfactual consumption moments across the state pairs formed by each state. For example, Alaska's consumption rises most by 29.8% with the reduction of trade costs across all the states whose mean increase in consumption is 10.3%. Meanwhile, the mean reduction in consumption volatility across states is 0.7%, 1.0%, and 0.3% respectively absent bilateral trade, migration, and financial frictions. For a risk-averse agent, lower consumption volatility implies higher lifetime utility. Therefore, the finding that eliminating the frictions reduces consumption fluctuations reiterates the significance of the three channels of risk sharing for improving welfare.

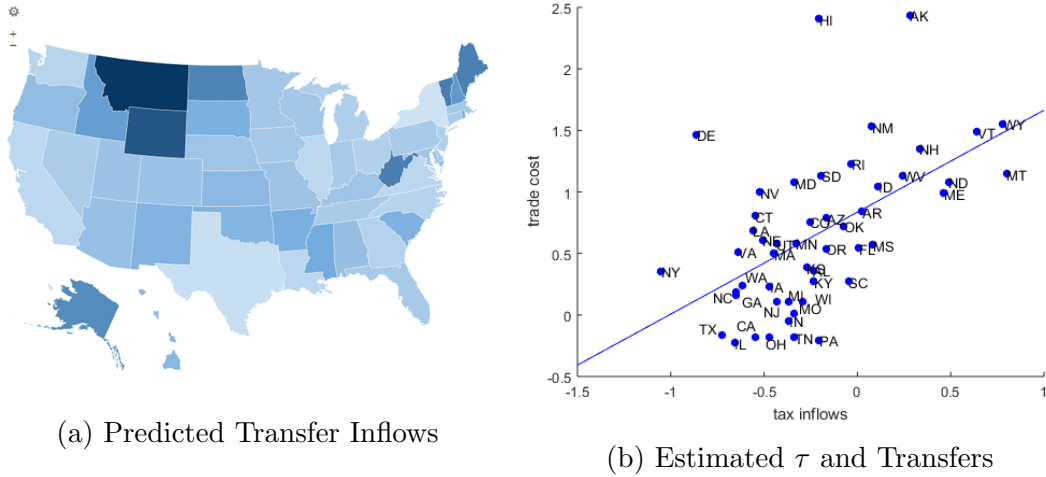
We use these counterfactual results to deliver policy implications. The spatial characteristics of frictions imply that lifting barriers in the channels of risk sharing is challenging due to geographic constraints. Nevertheless, macro policies can be introduced to alleviate the negative impacts of the frictions. In particular, fiscal transfers have been acknowledged as an important channel of risk sharing within a country. Redistribution of wealth from beneficiaries to victims of frictions can potentially undo the influences of frictions on the level and synchronization of consumption. On the modeling side, introducing optimal fiscal transfers T_i^* rewrites state i 's wealth constraint

$$\mathcal{W}_{i,t+1} = e^{-f_{\mathcal{L}i}} R_{\mathcal{L},t} \mathcal{W}_{i,t} + \sum_j^{\mathcal{I}} \alpha_{j,i,t} (e^{-f_{j^i}} R_{j,t} - e^{-f_{\mathcal{L}i}} R_{\mathcal{L},t}) + p_{i,t} \sum_{s \in \{T, NT\}} Y_{is,t} + T_{i,t}^* - P_{i,t} C_{i,t} - P_{Ii,t} I_{i,t}. \quad (32)$$

It is noteworthy that T_i^* is a supplementary transfer added to the existing transfers already reflected in state i 's calibrated wealth position. Under the new constraint with the additional T_i^* , households in state i choose their expenditure and make migration decisions based on the updated cross-state consumption differentials. Meanwhile, the portfolio of state i is re-constructed according to the risk-sharing needs given the new wealth position. Therefore, the design of fiscal policies considers all the endogenous changes of variables including their interactions in different channels of risk sharing.

To exemplify such policy analysis, we evaluate the optimal fiscal transfers that mitigate the impacts of trade cost on the level of consumption. The targeted moment for

Figure 8: Tax Transfers under Trade Costs



Note: Figure (a) plots the tax transfers as shares of a state’s GSP to achieve its level of consumption in the counterfactual situation absent trade costs. A darker color in the heatmap suggests more tax inflows. Figure (b) shows the positive relationship between the transfer and estimated trade costs, calculated as the geometric mean of inbound and outbound trade costs reported in table A.4, relative to Georgia and Ohio the median states in terms of output per capita.

the policy design is consumption per capita absent bilateral trade cost. For a state pair consisting of i and j , we solve for T_i^* and T_j^* as their transfer inflows. To keep the aggregate budget constraint of the federal government intact, the rest of the economy (ROE)’s transfer outflows will be the sum of T_i^* and T_j^* . We conduct the policy analysis for all the state pairs and, for cross-state comparison plot the median tax transfers across the state pairs formed by each state in figure 8. The model predicts that, states confronted with higher trade costs, such as Wyoming, Montana, and Alaska, should receive more tax transfers to alleviate the impacts of trade frictions on their consumption. In contrast, states that face lower trade costs, including New York, Texas, and California, should be net tax payers to achieve the counterfactual outcome. The general relationship between the predicted transfers and the estimated trade costs is positive.

This example shows that the quantitative model provides a useful framework for policy analyses. The framework is general enough to accommodate a rich set of targeted moments including the level, volatility, and covariance of macroeconomic variables. These policies that facilitate consumption risk sharing reduce both consumption volatility over time and consumption disparity across space.

4 Conclusion

This paper empirically and theoretically examines how bilateral economic exchanges shape the geographic pattern of consumption. In particular, we exploit the variation across US state pairs and evaluate the channels of consumption risk sharing including trade, migration, and finance. Quantitative assessment of the model provides both economic insights on how the channels interact to influence consumption and implications for macro policy aiming to reduce consumption inequality.

One extension of our real business cycle framework is to introduce the New Keynesian ingredients, as [Hazell et al. \(2022\)](#) reason that cross-state heterogeneity generates different slopes of the Phillips Curve and consequently creates welfare disparity in a monetary union. Incorporating nominal rigidity into the model allows for examining the influences of monetary policy on the transmission and propagation of economic shocks through disaggregate cross-state economic linkages studied in this paper.

Our framework focuses on the US cross-state analysis but it is general enough to be tailored to another setting such as the European Union with a high degree of bilateral exchanges in multiple channels. Moreover, it can be used to compare intra- and international linkages to diagnose the border effects of risk sharing proposed by [Devereux and Hnatkovska \(2020\)](#), so as to provide guidance for tariffs and exchange rate policies. Such policies which help to reduce consumption disparity both within and across country borders will yield important welfare implications for the world economy.

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Figure A.1: U.S. Map

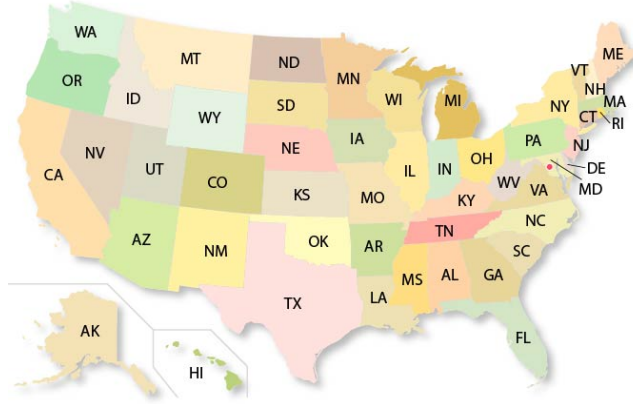


Table A.1: List of US States with Abbreviations

Name	Abbreviation	Name	Abbreviation	Name	Abbreviation	Name	Abbreviation	Name	Abbreviation
Alabama	AL	Hawaii	HI	Massachusetts	MA	New Mexico	NM	South Dakota	SD
Alaska	AK	Idaho	ID	Michigan	MI	New York	NY	Tennessee	TN
Arizona	AZ	Illinois	IL	Minnesota	MN	North Carolina	NC	Texas	TX
Arkansas	AR	Indiana	IN	Mississippi	MS	North Dakota	ND	Utah	UT
California	CA	Iowa	IA	Missouri	MO	Ohio	OH	Vermont	VT
Colorado	CO	Kansas	KS	Montana	MT	Oklahoma	OK	Virginia	VA
Connecticut	CT	Kentucky	KY	Nebraska	NE	Oregon	OR	Washington	WA
Delaware	DE	Louisiana	LA	Nevada	NV	Pennsylvania	PA	West Virginia	WV
Florida	FL	Maine	ME	New Hampshire	NH	Rhode Island	RI	Wisconsin	WI
Georgia	GA	Maryland	MD	New Jersey	NJ	South Carolina	SC	Wyoming	WY

Appendices

A Figures and Tables

Tables A.2 and A.3 report the results of two sets of robustness checks for the gravity model of risk sharing. First, we consider alternative data sources for state-level consumption and inflation, and for bilateral geographic distance. Second, we reconstruct measures of bilateral risk sharing after adjusting for additional time-series and cross-section variations (see the detailed description in the next paragraph). The results reported in the tables suggest that our finding about the comovement between geographic distance and consumption risk sharing remains robust.

When constructing alternative measures of bilateral risk sharing, first we consider demographic variables whose dynamics potentially shift consumption demand over time. These state-level variables (denoted as $X_{i,t}$) include average age, gender ratio, and education levels, whose data are obtained from the American Community Survey conducted by the Census Bureau. The estimation of risk sharing coefficients becomes

$$\Delta \log c_{it} - \Delta \log c_{jt} = \alpha_{ij} + \beta_{ij}(\Delta \log y_{it} - \Delta \log y_{jt}) + \mu_i X_{i,t} + \mu_j X_{j,t} + \epsilon_{ijt}. \quad (\text{A1})$$

Second, we adjust for states' distinct exposure to aggregate risks when measuring bilateral

Table A.2: Spatial Pattern of Risk Sharing – Alternative Data Sources

Dep. Var.: $\hat{\beta}_{ij}$	A. Alternative Price		B. Alternative Consumption		C. Alternative Distance	
	(1)	(2)	(3)	(4)	(5)	(6)
$\log(d_{ij})$	0.119 *** (0.017)	0.176 *** (0.024)	0.041 *** (0.005)	0.050 *** (0.007)	0.151 *** (0.010)	0.211 *** (0.012)
Geographic Variables	N	Y	N	Y	N	Y
Political Dissimilarity	N	Y	N	Y	N	Y
Industrial Dissimilarity	N	Y	N	Y	N	Y
Observations	528	528	1225	1225	1225	1225
R^2	0.077	0.183	0.056	0.148	0.161	0.288

Robust standard errors in parentheses. *** significant at 1%. The dependent variable is the estimated risk sharing coefficient $\hat{\beta}_{ij}$. d_{ij} denotes the geographic distance between state i and j . Panel A uses the state-level CPI data by Hazell et. al. (2020), Panel B uses the BEA consumption data, and Panel C uses the shipment distance from the CFS. Geographic variables and political/industrial dissimilarity measures remain the same as in the baseline estimation (table 2).

risk sharing, as the difference in output growth between a pair of states in equation 1 may reflect the two states' heterogeneous exposure to national shocks. To address this potential mismeasurement of local output shocks, we first estimate β_i and β_j from

$$\Delta \log y_{it} = \alpha_i + \beta_i \Delta \log y_{US t} + \epsilon_{it}, \quad \Delta \log y_{jt} = \alpha_j + \beta_j \Delta \log y_{US t} + \epsilon_{jt}, \quad (\text{A2})$$

where $\Delta \log y_{US t}$ denotes the growth of log real per-capita output of the US, and hence β_i captures the impact of aggregate shocks on state i 's output. We then estimate β_{ij} from

$$\Delta \log c_{it} - \Delta \log c_{jt} = \alpha_{ij} + \beta_{ij} [(\Delta \log y_{it} - \beta_i \Delta \log y_{US t}) - (\Delta \log y_{jt} - \beta_j \Delta \log y_{US t})] + \epsilon_{ijt}. \quad (\text{A3})$$

We also consider the bootstrap method for the potential finite sample bias from equation A2. Specifically, we draw a random sample with replacement (30 out of 43 years of sample) when running regression A3 to generate β_{ij} . When we regress the obtained β_{ij} on distance 1000 times for its estimate γ , we find the result to remain significant given the confidence interval as the 2.5% and 97.5% quantiles of the $\hat{\gamma}$ distribution.

Table A.3: Spatial Pattern of Risk Sharing – Alternative β

Dep. Var.: $\hat{\beta}_{ij}$	A. β_{ij} adjusted for demand shifters		B. β_{ij} adjusted for aggregate shocks	
	(1)	(2)	(3)	(4)
$\log(d_{ij})$	0.128 *** (0.013)	0.143 *** (0.017)	0.147 *** (0.010)	0.214 *** (0.012)
Geographic Variables	N	Y	N	Y
Political Dissimilarity	N	Y	N	Y
Industrial Dissimilarity	N	Y	N	Y
Observations	1225	1225	1225	1225
R^2	0.067	0.205	0.148	0.315

Robust standard errors in parentheses. *** significant at 1%. The dependent variable in panel A (B) is the estimated β_{ij} based on equation A1 (A3). d_{ij} denotes distance between i and j . Geographic variables and political/industrial dissimilarity measures remain the same as in the baseline estimation (table 2).

Table A.4: Estimated Frictions and Counterfactual Predictions by State

State	Panel (I). Estimated Frictions						Panel (II). Counterfactual Predictions				
	Trade Cost τ		Migration Cost d		Financial Cost f		Equilibrium Level \bar{c}		Volatility σ_c		
	Out(bound)	In(bound)	Out	In	Out	In	No τ	No d	No τ	No d	No f
AL	0.975	1.476	1.035	1.117	0.493	0.592	1.058	0.958	1.015	0.974	1.007
AK	3.136	3.643	0.888	1.146	30.850	54.888	1.298	0.955	0.908	0.969	0.981
AZ	1.561	1.410	0.996	0.974	1.403	1.281	1.072	0.985	0.999	0.993	1.000
AR	1.007	2.296	1.002	1.115	1.562	0.754	1.161	0.981	1.068	1.015	1.000
CA	1.845	0.452	1.018	0.858	0.930	0.568	1.033	1.044	0.987	1.047	1.018
CO	1.406	1.520	0.934	0.966	1.379	1.864	1.067	0.978	1.036	1.009	1.049
CT	1.478	1.513	1.033	1.165	5.474	3.356	1.092	0.998	0.939	1.006	0.993
DE	1.536	2.822	1.069	1.175	80.416	72.842	1.202	0.967	0.816	0.970	0.998
FL	1.731	0.994	1.007	0.821	1.277	7.177	0.979	1.032	0.998	0.995	1.003
GA	1.057	1.113	0.970	0.973	1.292	1.393	1.026	0.983	0.966	0.971	1.003
HI	2.710	4.099	0.980	1.086	6.792	9.723	1.094	0.977	0.953	0.979	1.000
ID	1.045	2.719	1.019	1.159	3.249	5.006	1.200	0.931	1.036	0.987	1.002
IL	1.111	0.719	0.988	0.983	0.750	0.672	1.009	0.978	0.972	0.994	1.002
IN	0.917	1.042	0.999	1.044	3.381	2.784	1.050	0.943	0.970	0.982	0.958
IA	0.646	1.952	1.005	1.080	7.757	4.730	1.064	0.947	0.879	0.967	1.010
KS	0.702	2.099	0.978	1.060	3.390	2.600	1.059	0.962	0.959	0.963	0.986
KY	0.884	1.483	1.000	1.074	7.201	6.939	1.051	0.948	0.966	0.983	0.998
LA	1.151	1.729	1.030	1.105	2.384	3.223	1.075	0.968	0.897	1.002	0.991
ME	1.128	2.384	1.019	1.181	0.002	2.119	1.165	0.939	1.156	0.971	1.000
MD	1.766	1.660	1.029	1.058	9.218	3.651	1.070	0.974	1.003	0.990	1.001
MA	1.374	1.200	1.005	1.064	3.732	3.272	1.036	0.980	0.958	0.988	1.004
MI	0.938	1.189	1.030	1.038	2.645	4.517	1.021	0.993	0.958	0.999	1.005
MN	1.150	1.555	1.025	1.076	1.414	0.780	1.082	0.972	0.997	0.966	1.006
MS	0.865	2.047	1.014	1.153	2.014	6.122	1.127	0.954	1.033	0.990	0.991
MO	0.921	1.101	1.008	1.032	1.119	0.827	1.071	0.974	1.022	0.992	0.990
MT	1.291	2.440	0.975	1.152	0.022	0.201	1.213	0.906	1.112	0.985	1.000
NE	1.082	1.695	1.025	1.167	14.183	14.576	1.136	0.957	0.910	1.017	0.965
NV	1.319	2.052	0.980	1.086	1.060	1.493	1.097	0.968	0.979	0.985	1.000
NH	1.522	2.535	1.013	1.193	1.580	3.732	1.250	0.983	1.106	0.992	1.000
NJ	1.012	1.104	1.018	1.068	0.899	0.883	1.002	0.976	0.946	0.990	1.001
NM	2.197	2.103	0.998	1.128	8.109	14.685	1.221	0.988	0.969	1.018	0.996
NY	2.122	0.673	1.074	0.977	8.658	7.305	1.027	1.018	0.956	1.038	1.000
NC	0.901	1.339	1.018	0.957	0.646	0.924	1.024	0.989	0.975	1.004	0.969
ND	0.910	3.245	0.984	1.177	0.735	5.364	1.263	0.919	1.041	1.032	1.000
OH	0.943	0.887	1.030	1.027	0.708	0.607	1.014	0.965	0.957	1.010	1.010
OK	1.077	1.913	1.036	1.113	1.754	0.808	1.080	0.964	0.997	0.984	0.981
OR	1.083	1.585	1.027	1.128	3.052	3.060	1.070	0.952	0.982	0.977	0.959
PA	1.070	0.762	1.021	1.032	0.216	0.308	1.001	0.974	1.012	0.987	1.000
RI	1.081	3.156	1.068	1.213	0.690	1.087	1.197	0.946	1.117	0.984	1.007
SC	0.983	1.334	1.003	1.034	0.283	0.633	1.091	0.959	1.080	0.965	1.003
SD	0.909	3.413	0.997	1.162	11.196	11.012	1.245	0.903	0.901	0.928	0.951
TN	0.884	0.942	0.978	0.995	1.836	2.071	1.075	0.955	0.999	0.981	1.000
TX	1.236	0.690	0.999	0.849	1.249	1.208	0.964	0.993	0.932	1.031	1.032
UT	0.951	1.873	1.013	1.125	2.752	3.114	1.135	0.962	0.971	0.979	0.995
VT	1.082	4.098	1.035	1.214	0.023	0.374	1.329	0.909	1.193	0.985	1.000
VA	1.252	1.335	0.997	0.976	2.416	2.006	1.001	0.979	0.999	0.994	1.000
WA	0.954	1.330	1.018	1.006	1.188	1.222	1.033	0.989	0.923	1.005	1.001
WV	1.070	2.900	1.084	1.201	0.308	14.961	1.093	0.941	1.070	1.001	1.004
WI	1.166	0.957	1.037	1.082	0.926	0.692	1.072	0.959	1.030	0.983	0.998
WY	1.490	3.177	0.932	1.157	0.018	0.566	1.356	0.927	1.018	0.962	1.000
Mean	1.253	1.835	1.009	1.074	4.893	5.891	1.103	0.966	0.993	0.990	0.997
Median	1.082	1.570	1.013	1.081	1.488	2.095	1.073	0.968	0.984	0.988	0.999

Panel (I) presents the normalized trade, migration, and financial costs averaged across state pairs for each state. We first calculate both inbound and outbound frictions averaged across $\mathcal{I} - 1$ pairs a state i forms: ($x_i^{ex} = \text{mean}(x_{ij})$, $x_i^{in} = \text{mean}(x_{ji})$, $j \neq i \in [1, \mathcal{I}]$, $x \in \{\tau, d, f\}$). We then normalize the average friction of Georgia and Ohio, the median states in terms of output per capita, to 1 in each channel: $x_{GA,OH}^{ex} = x_{GA,OH}^{in} = 1$, and report the ratio of state-level frictions relative to the median states' in the table for cross-state comparison. Panel (II) presents each state's median counterfactual steady-state level and volatility of consumption across its state pairs, as a ratio to the values in original case with frictions calibrated to the data. Counterfactual scenarios include the cases absent bilateral trade costs (τ), migration costs (d), and financial frictions (f).

B Data

B.1 State-level output, consumption, and price

The US Bureau of Economic Analysis (BEA) reports the real GDP by state (GSP) since 1977, with data from 1977-1997 reported in the Standard Industrial Classification (SIC) and those from 1997-2019 in the North American Industry Classification (NAICS). To address this discontinuity, we first calculate the annual growth rate based on the SIC-based real GSP, and then reconstruct the time series of real GSP from 1977 to 1997 using this annual growth rate and the NAICS-based real GSP in 1997.

The nominal consumption data from the BEA are only available after 1997, which is not ideal for our risk-sharing analysis over a long horizon. Therefore, we follow [Asdrubali et al. \(1996\)](#)'s method of constructing state-level private consumption by rescaling state-level retail sales by the country-level ratio of private consumption to retail sales, both obtained from the BEA. To convert nominal to real consumption, we use the state-level inflation series constructed by [Nakamura and Steinsson \(2014\)](#) over the period from 1966 to 2008. They obtain the inflation series from 1966 to 1995 from [Del Negro \(1998\)](#), who combines the BLS regional inflation data and cost-of-living estimates from the American Chamber of Commerce Realtors Association (ACCRA). For the estimates between 1995 and 2008, they multiply a population-weighted average of cost-of-living indices from the ACCRA across states with the US aggregate CPI. After 2008, we use the Regional Price Parities (RPP) from the BEA that measure price differences within the United States. RPP is a weighted average of the price level of goods and services for the average consumer in one geographic region compared to all other regions in the US. We merge these data to construct a state-level CPI index for 1966-2019, using which we deflate the nominal consumption data to calculate real consumption at the state level.

We also use alternative data sources to verify the robustness of the gravity model. Table [A.2](#) Panel A uses state-level inflation from [Hazell et al. \(2022\)](#) who construct CPI with micro data gathered by the BLS from 1978 to 2017. Panel B uses only the recent BEA data of consumption expenditure and real GSP between 1997 and 2018.

B.2 Bilateral trade and migration flows

The Commodity Flow Survey (CFS) is conducted every five years by the Census Bureau in partnership with the Department of Transportation. The survey provides detailed information on commodity flows within the US, including the type of commodities shipped, origin and destination, value and weight, and mode of transport. There are six waves of surveys so far (1993, 1997, 2002, 2007, 2012, 2017).

State-to-state migration data are based on year-to-year address changes reported on individual income tax returns filed with the Internal Revenue Service (IRS). Specifically, we use the reported number of returns filed every year to track migration across states. The data are available for filing years 1991 through 2019.

B.3 State-level productivity

We estimate the state-level total factor productivity (TFP) as the Solow residual from

$$\log(A_{i,t}) = \log(Y_{i,t}) - \alpha \log(K_{i,t}) - (1 - \alpha) \log(L_{i,t}), \quad (\text{A4})$$

where $Y_{i,t}$, $K_{i,t}$, and $L_{i,t}$ are output, capital, and labor in state i at time t respectively, while α denotes capital share in production. We estimate $1 - \alpha$ to be 0.59 by dividing the labor earnings by the economic output based on the BEA data.²⁰ Moreover, we use the GSP and employment data reported by the BEA for $Y_{i,t}$ and $K_{i,t}$ over the period 1977-2019 for the estimation.

We construct the estimates for state-level capital stock following [Garofalo and Yamarik \(2002\)](#). Namely, we apportion the national private capital stock, to states using sectoral income data from the BEA: For each two-digit NAICS industry

$$K_{i,t}^s = \left(\frac{Y_{i,t}^s}{Y_{US,t}^s} \right) K_{US,t}^s, \quad (\text{A5})$$

where $K_{i,t}^s$ ($Y_{i,t}^s$) refers to capital (output) of industry s in state i at time t , while $K_{US,t}^s$ ($Y_{US,t}^s$) represents country-level capital (output). Each state's capital stock estimate, $K_{i,t}$, is then the sum of sectoral-level capital stock:

$$K_{i,t} = \sum_{s=1}^S K_{i,t}^s. \quad (\text{A6})$$

After obtaining the values of all the variables in equation [A4](#), we calculate TFP with which we subsequently estimate the joint productivity process across states.

C Details of the Quantitative Model

This section provides details of the quantitative model for three-state analysis. Section [C.1](#) discusses the calibration strategy. Section [C.2](#) explains the solution to the portfolio choice problem in a trilateral framework.

C.1 Calibration

Many common parameters and state-specific variables are calibrated in the same way as in the two-economy model from section [3.2](#). The variables of the rest of the economy (ROE) from a state-pair's perspective, denoted with asterisks below, will be the sum of all the \mathcal{I} states' variables minus the state-pair's. Therefore, ROE's productivity for i and

²⁰The BEA reports the data of labor earning(SAINC5), which consists of compensation of employees and proprietors' income with inventory valuation adjustment and capital consumption adjustment.

j at time t is computed from

$$\begin{aligned}
\log(A_t^{ij*}) &= \log(Y_t^{ij*}) - \alpha \log(K_t^{ij*}) - (1 - \alpha) \log(L_t^{ij*}) \\
&\equiv \log\left(\sum_i^{\mathcal{I}} Y_{i,t} - Y_{i,t} - Y_{j,t}\right) - \alpha \log\left(\sum_i^{\mathcal{I}} K_{i,t} - K_{i,t} - K_{j,t}\right) \\
&\quad - (1 - \alpha) \log\left(\sum_i^{\mathcal{I}} L_{i,t} - L_{i,t} - L_{j,t}\right).
\end{aligned} \tag{A7}$$

We then obtain the variance-covariance matrix (Σ) of these three states' productivity assuming the annual persistence of productivity is 0.72, which is estimated from the U.S. country-level Solow residual.

Another distinct feature of this asymmetric framework is that each state may not run a balanced budget in the equilibrium. To this end, we collect the data on state-level output and expenditure (defined as the sum of consumption and investment), whose difference represents the wealth position of the economy that also includes any fiscal transfer received by it. ROE's wealth position will be the sum of all the states' positions minus the positions of the state-pair under examination.

We now proceed to discuss the calibration strategies for bilateral frictions in the trilateral framework. Our calibration is based on the sample period from 1997 to 2017. The sample selection is largely driven by the availability of the CFS trade data. We use the time-averaged state-level population, net asset positions, trade and migration flows as the steady-state values of those variables when estimating and solving the model. There are three economies numbered 1, 2, 3 with 1 and 2 representing the pair of states being studied and 3 representing ROE. The three economies encounter a set of six bilateral frictions in each of the trade, migration, and finance channels

$$\{x_{12}, x_{13}, x_{23}, x_{21}, x_{31}, x_{32}\}, \quad x \in \{\tau, d, f\}. \tag{A8}$$

In terms of trade and migration costs, we estimate them simultaneously to ensure that the model-predicted bilateral migration and trade linkages match those from the IRS and CFS data. The estimation procedure is similar to that in section 3.2: Step 1, we start with an initial guess for the combination of migration and trade costs. Step 2, we solve for wage rates and labor hours given the frictions that satisfy the labor market clearing condition. Step 3, we calculate the corresponding trade and migration shares to the wages solved earlier. Step 4, we repeat the previous steps until the trade and migration shares converge to the empirical moments.

After characterizing the real side of the model, we calibrate frictions in the financial channel to the pattern of consumption comovement across economies. Specifically, we estimate the coefficients of consumption risk sharing among the three economies with the same data and method as in the empirical section

$$\beta = [\beta_{12}, \beta_{13}, \beta_{23}], \tag{A9}$$

Table A.5: Estimated Financial Frictions and Banking Linkage

Dep. Var: Est. Frictions $\log(\hat{f}_{ij})$	(1)	(2)
Branches	-5.7e-04*** (1.1e-04)	
Deposits		-6.8e-09*** (1.6e-09)
Observations	2442	2442
R^2	0.001	0.001

Robust standard errors in parentheses, *** significant at 1%. The independent variable is the estimated bilateral financial friction between states i and j . Dependent variables include the number of bank branches, and the dollar amount of deposits collected by financial institutions, located in i and headquartered in j . Estimated frictions are missing for few pairs because the eigenvalues computed at the steady state of the model for those pairs do not satisfy the Blanchard and Kahn (1980) condition to guarantee the existence of a unique solution.

and use the coefficients as targeted moments to estimate bilateral frictions. Appendix C.2 outlines the technical details of the portfolio choice problem in this trilateral framework. The algorithm is slightly modified from that in section 3.2: First, we obtain the coefficient matrices, including R_1, R_2, D_1, D_2 in equations A20-A21 necessary to solve the portfolio choice problem from the first-order dynamics of the model. Second, we solve for asset holdings under which the model-implied risk-sharing coefficients β match those estimated from the data. To simplify our computation in this step, we assume a state's holding of ROE's assets is the same whose baseline weight in the portfolio is one-half but the state can choose the remaining composition between its own and pair partner's assets under risk-sharing motives. Third, we plug the calibrated asset positions in the portfolio determination equation (equation A16) to compute financial frictions.

Among the three frictions, we are particularly interested in testing whether our estimated financial frictions are reasonable. To this end, we collect the Federal Deposit Insurance Corporation (FDIC) bank statistics, which list branch locations and their reported deposits. States i and j are deemed to exhibit stronger financial ties when banks headquartered in i open more local branches in j or collect more deposits from branches located in j . Therefore, we compile this information of all the FDIC-insured institutions and explore its consistency with financial frictions \hat{f}_{ij} . Based on the results presented in table A.5, an increase of one thousand branches or one billion deposits collected by institutions, located in i and headquartered in j , is associated with a decrease of .57% or 6.8% estimated financial frictions (\hat{f}_{ij}) respectively. This analysis provides external validity for our estimates: Financial frictions estimated from the consumption data are consistent with empirical evidence from the banking sector. That said, as discussed in the main text, the estimated financial frictions reflect the deviation of allocation from complete markets and therefore take many other forms beyond the banking sector.

C.2 Portfolio Choice in Trilateral Framework

This section describes the portfolio choice problem in a framework with three economies numbered $i = 1, 2, 3$. Each economy's financial asset, which is its claims to capital income net of investment expenditure, can be traded in an integrated financial market. Nevertheless, there are bilateral financial frictions modeled as transaction costs f_{ij} on returns R_i when j holds assets from i . These second-order frictions appear in the Euler equations of the three economies

$$\begin{aligned} E_t\left[\frac{U'(c_{1,t+1})}{P_{1,t+1}}R_{1,t+1}\right] &= E_t\left[\frac{U'(c_{1,t+1})}{P_{1,t+1}}e^{-f_{21}}R_{2,t+1}\right] = E_t\left[\frac{U'(c_{1,t+1})}{P_{1,t+1}}e^{-f_{31}}R_{3,t+1}\right], \\ E_t\left[\frac{U'(c_{2,t+1})}{P_{2,t+1}}R_{2,t+1}\right] &= E_t\left[\frac{U'(c_{2,t+1})}{P_{2,t+1}}e^{-f_{12}}R_{1,t+1}\right] = E_t\left[\frac{U'(c_{2,t+1})}{P_{2,t+1}}e^{-f_{32}}R_{3,t+1}\right], \\ E_t\left[\frac{U'(c_{3,t+1})}{P_{3,t+1}}R_{3,t+1}\right] &= E_t\left[\frac{U'(c_{3,t+1})}{P_{3,t+1}}e^{-f_{13}}R_{1,t+1}\right] = E_t\left[\frac{U'(c_{3,t+1})}{P_{3,t+1}}e^{-f_{23}}R_{2,t+1}\right]. \end{aligned} \quad (\text{A10})$$

We derive portfolios with [Devereux and Sutherland \(2011\)](#)'s method by evaluating these Euler equations. First we assume assets from economy 3 to be a numeraire asset and denote the vector of excess returns to the other assets as R_x :

$$\hat{R}'_{x,t} = [\hat{R}_{1,t} - \hat{R}_{3,t}, \hat{R}_{2,t} - \hat{R}_{3,t}], \quad (\text{A11})$$

where \hat{y}_t represents the log-deviation of any variable y from its steady state at t . Next we evaluate the second-order Taylor expansion of the Euler equations as

$$\begin{aligned} E_t[\hat{R}_{x,t+1} + \frac{1}{2}\hat{R}_{x,t+1}^2 - (\sigma\hat{c}_{1,t+1} + \hat{P}_{1,t+1})\hat{R}_{x,t+1}] &= -\frac{1}{2}\begin{bmatrix} f_{31} \\ f_{31} - f_{21} \end{bmatrix} + \mathcal{O}(\epsilon^3), \\ E_t[\hat{R}_{x,t+1} + \frac{1}{2}\hat{R}_{x,t+1}^2 - (\sigma\hat{c}_{2,t+1} + \hat{P}_{2,t+1})\hat{R}_{x,t+1}] &= -\frac{1}{2}\begin{bmatrix} f_{32} - f_{12} \\ f_{32} \end{bmatrix} + \mathcal{O}(\epsilon^3), \\ E_t[\hat{R}_{x,t+1} + \frac{1}{2}\hat{R}_{x,t+1}^2 - (\sigma\hat{c}_{3,t+1} + \hat{P}_{3,t+1})\hat{R}_{x,t+1}] &= -\frac{1}{2}\begin{bmatrix} -f_{13} \\ -f_{23} \end{bmatrix} + \mathcal{O}(\epsilon^3). \end{aligned} \quad (\text{A12})$$

where $\hat{R}_{x,t+1}^2$ denotes differences in squared changes of returns

$$\hat{R}_{x,t+1}^2 = [\hat{R}_{1,t+1}^2 - \hat{R}_{3,t+1}^2, \hat{R}_{2,t+1}^2 - \hat{R}_{3,t+1}^2]. \quad (\text{A13})$$

On the right-hand side of equations [A12](#) are vectors of financial frictions each country incurs when holding assets from economies 1 and 2 relative to the frictions associated with its holding assets from economy 3. Plus, the last term $\mathcal{O}(\epsilon^3)$ captures all terms of order higher than two. Taking the difference among equations [A12](#) yields

$$\begin{aligned} E_t[(\hat{c}_{12,t+1} + \frac{\hat{P}_{12,t+1}}{\sigma})\hat{R}_{x,t+1}] &= \frac{1}{2\sigma}\begin{bmatrix} f_{31} - f_{32} + f_{12} \\ f_{31} - f_{21} - f_{32} \end{bmatrix} + \mathcal{O}(\epsilon^3), \\ E_t[(\hat{c}_{13,t+1} + \frac{\hat{P}_{13,t+1}}{\sigma})\hat{R}_{x,t+1}] &= \frac{1}{2\sigma}\begin{bmatrix} f_{13} + f_{31} \\ f_{31} - f_{21} + f_{23} \end{bmatrix} + \mathcal{O}(\epsilon^3), \\ E_t[(\hat{c}_{23,t+1} + \frac{\hat{P}_{23,t+1}}{\sigma})\hat{R}_{x,t+1}] &= \frac{1}{2\sigma}\begin{bmatrix} f_{32} - f_{12} + f_{13} \\ f_{23} + f_{32} \end{bmatrix} + \mathcal{O}(\epsilon^3), \end{aligned} \quad (\text{A14})$$

where $c_{ij,t} = \frac{c_{i,t}}{c_{j,t}}$ and $P_{ij,t} = \frac{P_{i,t}}{P_{j,t}}$ denote cross-region consumption and price ratios of i to j , which constitute a vector of price-adjusted consumption differential defined as

$$\frac{\hat{c}p'_t}{\sigma} = [\hat{c}_{12,t} + \frac{\hat{P}_{12,t}}{\sigma}, \hat{c}_{13,t} + \frac{\hat{P}_{13,t}}{\sigma}, \hat{c}_{23,t} + \frac{\hat{P}_{23,t}}{\sigma}]. \quad (\text{A15})$$

Equations A14 can therefore be re-written in the vector form as

$$E_t[\hat{c}p'_t \hat{R}'_{x,t+1}] = \frac{\mathcal{F}}{2} \equiv \frac{1}{2} \begin{bmatrix} f_{31} - f_{32} + f_{12} & f_{31} - f_{21} - f_{32} \\ f_{13} + f_{31} & f_{31} - f_{21} + f_{23} \\ f_{32} - f_{12} + f_{13} & f_{23} + f_{32} \end{bmatrix} + \mathcal{O}(\epsilon^3). \quad (\text{A16})$$

On the left hand side of this portfolio determination equation are two components: inflation-adjusted consumption differential $\hat{c}p$ and excess financial returns \hat{R}_x . Both components can be expressed in terms of region-specific innovations

$$\epsilon'_t = [\epsilon_{1,t}, \epsilon_{2,t}, \epsilon_{3,t}], \quad (\text{A17})$$

whose coefficients, as a function of portfolio choice, need to satisfy equation A16 in the equilibrium of the model. Let $\alpha_{i,j}$ represent j 's holding of asset i , then the unknown portfolio matrix scaled by the discount factor β and the region's steady-state output \bar{Y} to be solved in this three-economy framework is

$$\tilde{\alpha} = \frac{1}{\beta \bar{Y}} \begin{bmatrix} \alpha_{1,1} & \alpha_{1,2} \\ \alpha_{2,1} & \alpha_{2,2} \end{bmatrix}, \quad (\text{A18})$$

while the remaining holdings $\alpha_{3,j}$ and $\alpha_{i,3}$ can be recovered from each region's budget constraint and asset market clearing condition respectively. Given the portfolio arrangement, excess portfolio return is defined as

$$\xi_t = \tilde{\alpha}' \hat{R}_{x,t}. \quad (\text{A19})$$

Region-specific productivity shocks ϵ_t affect the two components in equation A16 both directly and indirectly through ξ_t :

$$\hat{c}p_{t+1} = D_1 \xi_{t+1} + D_2 \epsilon_{t+1} + D_3 z_{t+1} + \mathcal{O}(\epsilon^2), \quad (\text{A20})$$

$$\hat{R}_{x,t+1} = R_1 \xi_{t+1} + R_2 \epsilon_{t+1} + \mathcal{O}(\epsilon^2), \quad (\text{A21})$$

where R_1, R_2, D_1, D_2, D_3 are the coefficient matrices extracted from the first-order conditions of the model. R_1 and D_1 capture the response of the two components (consumption differential and excess asset returns) to excess portfolio returns; R_2 and D_2 capture their response to productivity shocks; and D_3 are their response to other state variables in the model summarized by z . In addition, using $\xi_{t+1} = \tilde{\alpha}' \hat{R}_{x,t+1}$ allows us to express ξ_{t+1} ,

$\hat{c}p_{t+1}$, and $\hat{R}_{x,t+1}$ in terms of ϵ_{t+1} only:

$$\xi_{t+1} = \tilde{H}\epsilon_{t+1}, \quad \text{where} \quad \tilde{H} = \frac{\tilde{\alpha}'R_2}{1 - \tilde{\alpha}'R_1}; \quad (\text{A22})$$

$$\hat{c}p_{t+1} = \tilde{D}\epsilon_{t+1} + D_3z_{t+1} + \mathcal{O}(\epsilon^2), \quad \text{where} \quad \tilde{D} = D_1\tilde{H} + D_2. \quad (\text{A23})$$

$$\hat{R}_{x,t+1} = \tilde{R}\epsilon_{t+1} + \mathcal{O}(\epsilon^2), \quad \text{where} \quad \tilde{R} = R_1\tilde{H} + R_2. \quad (\text{A24})$$

Now that we have examined the two components in equation A16 separately as functions of innovations ϵ_{t+1} , we can multiply them to evaluate the portfolio determination condition:

$$E_t[\hat{c}p_{t+1}\hat{R}'_{x,t+1}] = \tilde{D}\Sigma\tilde{R}' = \frac{\mathcal{F}}{2}. \quad (\text{A25})$$

In terms of calibration, we follow the steps below to numerically estimate bilateral financial frictions f_{ij} . First, we extract coefficient matrices R_1, R_2, D_1, D_2 , and the response of the relative output differential $\hat{y}_{ij} = \hat{y}_i - \hat{y}_j$ to shocks from the first order conditions in the model. In particular, we take the first order derivative of output differential to productivity shocks

$$Dy = \frac{\partial y_{ij}}{\partial \epsilon}, \quad (\text{A26})$$

where ϵ is the vector of productivity shocks defined in A17. We use the same method to capture the response of the relative consumption differential $\hat{c}_{ij} = \hat{c}_i - \hat{c}_j$ to shocks

$$Dc = \frac{\partial c_{ij}}{\partial \epsilon}, \quad (\text{A27})$$

which based on equation A23 is influenced by portfolio choice $\tilde{\alpha}$ from A18 together with coefficient matrices R_1, R_2, D_1, D_2 calculated earlier. The coefficient of consumption risk sharing $\hat{\beta}_{ij}$ can therefore be approximated as the differential between Dc and Dy in response to productivity shocks. After we compute $\hat{\beta}_{ij}$ for each productivity shock following the steps above using the first-order dynamics of the model, we take the mean value of $\hat{\beta}_{ij}$ across shocks to get a state-pair's overall consumption risk sharing and compare it with the coefficient estimated with the method from the empirical section which serves as a targeted moment. We loop over different portfolios $\tilde{\alpha}$ until the model-predicted coefficient of risk sharing matches its empirical moment. After that, we plug the calibrated portfolio $\tilde{\alpha}$ in \tilde{D} and \tilde{R} of equation A25 to find matrix \mathcal{F} . Lastly, we recover bilateral financial frictions from this matrix of financial frictions based on equation A16.