General Equilibrium Analysis of Multinational Financial and Trade Linkages

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Motivation

Existing Literature

- Few multi-country structural models with both trade and finance
  - Trade literature: takes asset positions as exogenous
  - Macro literature: studies a small number of countries
- General equilibrium effects hence not fully captured

This Paper

- Combines macro/trade methodological breakthroughs
- Solves for countries' multilateral linkages in both channels
- Delivers policy implications with comparative statics analysis

Why Need A Unified Framework with Trade and Finance?

Questions not fully answered by existing literature

Trade Influences Finance

- How do cross-country input-output linkages draw the map of global capital allocation?
- How might a trade war affect countries' optimal asset positions?

Finance Influences Trade

- How would international trade respond to regional financial integration?
- How does China's financial liberalization affect its bilateral trade ties?

Relation to Literature

Portfolio choice solution method in DSGE framework (DSTW)

- Literature: Devereux and Sutherland 2011, Tille and van Wincoop 2010
- Idea: 2nd-order approx of Euler eqn + 1st-order approx of others $\Rightarrow$ steady-state (s.s.) portfolio
- Limitation: portfolios are solved around a fixed s.s. of the real economy

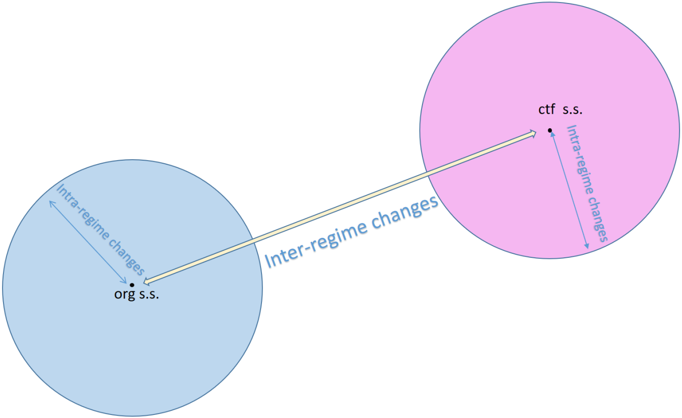
Exact hat algebra from trade literature (DEK)

- Literature: Dekle, Eaton and Kortum 2007
- Idea: characterize variables' changes instead of levels $\hat{X} = \frac{X^{ctf}}{X^{org}}$
- Limitation: finance is exogenously taken from data or endogenously derived in extreme cases (financial autarky or complete markets)

Contribution of this paper

- To trade literature: endogenizes financial allocation under intertemporal utility maximization + financial friction
- To macro literature: solves multinational financial and trade linkages for policy analysis with a new approach

Main Idea of the New Approach



- Policies both shift locations of s.s. and affect how variables behave around s.s.
- 2nd-moments around s.s. matter for portfolios solved by DSTW's method
- Asset positions influence the shift of s.s. solved by DEK's exact hat algebra
- GE characterized by a joint fixed point problem of financial and real variables

Relation to Literature (Continued)

Asset home bias in open economy macro (solved by DSTW method)

- Coeurdacier and Rey (2013), Heathcote and Perri (2013), Coeurdacier and Gourinchas (2016), Bergin and Pyun (2016), Hu (2020, 2023)
  - Solves portfolio under given trade/output structure
- This paper: Examines two-way interactions of trade and finance
  - Trade influences portfolio through risk sharing patterns
  - Financial allocation shifts demand in goods markets

Alternative portfolio solution techniques

- Literature: Asset demand system by Liu et al. (2022), Rational inattention logit demand by Pellegrino et al. (2022)
- Strengths of DSTW's method:
  - Doesn't need separate utility fn for intratemporal financial allocation
  - Portfolios driven by endogenous 2nd-moments capture mechanisms including risk sharing, risk hedging, and risk diversification
  - Offers wide macro applications with compatibility with DSGE models

Presentation Outline

Goods Market — Eaton-Kortum model

- A single tradable sector with intermediate goods

$$Q_{i,t} = \int_0^1 [q_{iu,t}(u)^{\frac{\epsilon-1}{\epsilon}} du]^{\frac{\epsilon}{\epsilon-1}}$$

- Productivity is the product of a variety-specific component drawn from a Fréchet distribution and a country-level component
- Country-level productivity $T_{i,t}$ follows an AR(1) subject to shocks with cross-country covariance Σ_T around mean $\bar{T}_i$

$$T_{i,t} = \rho T_{i,t-1} + (1-\rho)\bar{T}_i + \epsilon_{i,t}$$

- Production uses labor and capital with wage $w_{i,t}$ and rent $r_{i,t}$, and final goods as input with price $P_{i,t}$
- Bilateral trade shares given iceberg trade cost τ_{ij} from i to j

$$\pi_{ij,t} = \frac{T_{i,t}[\tau_{ij}(r_{i,t}^{\mu} w_{i,t}^{1-\mu})^{\eta} P_{i,t}^{1-\eta}]^{-\theta}}{\Phi_{j,t}}, \quad \Phi_{j,t} = \sum_{k=1}^{I} T_{k,t}[\tau_{kj}(r_{k,t}^{\mu} w_{k,t}^{1-\mu})^{\eta} P_{k,t}^{1-\eta}]^{-\theta}$$

- Goods market clearing:
$Y_{i,t} = \sum_{j=1}^{I} \pi_{ij,t} X_{j,t}$, where $X_{j,t} = (1-\eta)Y_{j,t} + P_{j,t}(C_{j,t} + IV_{j,t})$

Households

- A representative household in each country to maximize lifetime utility

$$\max E_0 \sum_{t=0}^{\infty} \nu_t \frac{C_{i,t}^{1-\gamma}}{1-\gamma}.$$

- Endogenous discount factor $\beta(C_{i,t}) = \omega_i C_{i,t}^{-\psi}$ following DS

$$\nu_0 = 1, \nu_{t+1} = \nu_t \beta(C_{i,t})$$

  to ensure stationary wealth distribution in incomplete markets
- Euler equation for capital investment

$$C_{i,t}^{\psi-\gamma} = \omega_i E_t[\frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}}((1-\delta)P_{i,t+1} + \frac{\partial Y_{i,t+1}}{\partial K_{i,t+1}})]$$

- Law of motion for physical capital

$$K_{i,t+1} = (1-\delta)K_{i,t} + IV_{i,t}$$

Asset Market

- Countries issue equities as claims to capital income with dividends ($d_{i,t}$), prices ($q_{i,t}$), and returns ($R_{i,t}$) following asset home bias lit

$$d_{i,t} = \eta\mu Y_{i,t} - P_{i,t} IV_{i,t}, \qquad R_{i,t+1} = \frac{d_{i,t+1} + q_{i,t+1}}{q_{i,t}}.$$

- Country i's wealth constraint

$$D_{i,t} = D_{i,t-1} e^{-f_{iI}} R_{I,t} + \sum_{k=1}^{I-1} \alpha_{ik,t-1}(e^{-f_{ik}} R_{k,t} - e^{-f_{iI}} R_{I,t}) + Y_{i,t} - X_{i,t}.$$

- Country i's asset positions and asset market clearing condition

$$D_{i,t} = \sum_{k=1}^{I} \alpha_{ik,t}, \qquad \sum_{k=1}^{I} \alpha_{ki,t} = 0$$

- Euler equation for different assets

$$\frac{C_{i,t}^{\psi-\gamma}}{P_{i,t}} = \omega_i E_t[\frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}} R_{i,t+1}] = \omega_i E_t[\frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}} e^{-f_{ij}} R_{j,t+1}], \forall i, j \in \{1, ..., I\}.$$

  - Inter-temporal investment influenced by $\beta, \gamma, P_{i,t+1}, R_{j,t+1}$
  - Intra-temporal investment influenced by Σ_T, f_{ij}

Bilateral Financial Frictions

Households in i incur f_{ij} when repatriating returns on asset j

Potential Determinants

- Worldwide factors including global financial liquidity
- Country-specific factors including capital account openness
- Pair-specific factors including geographic distance, regional agreements

Functional Form

- Transaction costs on foreign returns (Heathcote and Perri (2004))
- Alternatively, information frictions (Okawa and van Wincoop (2012))
- 2nd-order (proportional to variance of shocks) to retain certainty equivalence to 1st-order approximation
- Bilateral portfolio weights are sufficient statistics for financial frictions in comparative statics analysis

Portfolio Choice

Solution method developed by DSTW with linearization of DSGE models

- Log-deviation of any variable A from s.s.

$$\widetilde{A}_t = \ln(\frac{A_t - \bar{A}}{\bar{A}})$$

- Log-deviation of any cross-country ratio from s.s.

$$\widetilde{B}_{i/j,t} = \widetilde{B}_{i,t} - \widetilde{B}_{j,t} \text{ for } B_{i/j,t} = \frac{B_{i,t}}{B_{j,t}}$$

- Euler equation of country i

$$E_t[\frac{U'(C_{i,t+1})}{P_{i,t+1}} e^{-f_{iI}} R_{I,t+1}] = E_t[\frac{U'(C_{i,t+1})}{P_{i,t+1}} e^{-f_{i1}} R_{1,t+1}] ... = E_t[\frac{U'(C_{i,t+1})}{P_{i,t+1}} e^{-f_{iI\text{-}1}} R_{I\text{-}1,t+1}]$$

- Second-order Taylor expansion

$$E_t[\widetilde{R}_{x,t+1} + \frac{1}{2}\widetilde{R}^2_{x,t+1} - \widetilde{R}_{x,t+1}(\gamma \widetilde{C}_{i,t+1} + \widetilde{P}_{i,t+1})] = -\frac{1}{2}F_i + \mathcal{O}(\epsilon^3)$$

- Vector of excess returns relative to the numeraire asset I

$$\widetilde{R}'_{x,t+1} = [\widetilde{R}_{1,t+1} - \widetilde{R}_{I,t+1}, \widetilde{R}_{2,t+1} - \widetilde{R}_{I,t+1}, ..., \widetilde{R}_{I\text{-}1,t+1} - \widetilde{R}_{I,t+1}],$$

Portfolio Choice

- Portfolio determination equation

$$E_t[\widetilde{C}^p_{i/I,t+1}\widetilde{R}'_{x,t+1}] = \frac{1}{2}F_{iI}$$

with consumption differential $\widetilde{C}^p_{i/I,t+1} = \gamma\widetilde{C}_{i/I,t+1} + \widetilde{P}_{i/I,t+1}$

- Matrix of financial frictions

$$F_{iI} = [f_{iI} - f_{II}] \times \text{ones}(I-1,1) - [f_{i1} - f_{I1}, f_{i2} - f_{I2}, ..., f_{iI\text{-}1} - f_{II\text{-}1}].$$

- Stack the equation vertically with each row representing a holder country to characterize global financial allocation

$$E_t[\widetilde{C}^p_{x,t+1}\widetilde{R}'_{x,t+1}] = \frac{1}{2}F + \mathcal{O}(\epsilon^3)$$

- Take the derivative of two LHS terms wrt productivity shocks and derive portfolios that support consumption allocations. ▸ Computation ▸ Calibration

Presentation Outline

Mechanisms for Portfolio Choice

- Asset home bias literature (surveyed by Coeurdacier and Rey 2013)

$$\bar{\alpha}_{ii} = \underbrace{\frac{1}{2}}_{\substack{\text{Risk sharing}\\ \text{(Diversification)}}} - \underbrace{\frac{1}{2}\frac{1-\mu}{\mu}\frac{cov(\widetilde{w}_{i/j}, \widetilde{R}_{i/j})}{var(\widetilde{R}_{i/j})}}_{\text{Hedging labor income risk}} + \underbrace{\frac{1}{2}\frac{1-1/\gamma}{\mu}\frac{cov(\widetilde{P}_{i/j}, \widetilde{R}_{i/j})}{var(\widetilde{R}_{i/j})}}_{\text{Hedging RER risk}}.$$

- Portfolio should be diversified for risk sharing (Lucas 1982)
- Portfolio should tilt toward assets whose income
  - decreases with domestic labor income
  - increases with real exchange rate (RER)

Extensions made in this paper

- complete $\Rightarrow$ incomplete markets: 2nd-moments including asset covariances matter
- 2 $\Rightarrow$ 43 countries: individual foreign asset returns less correlated with domestic fundamentals

Disentangling Mechanisms for Portfolio Choice

- General portfolio equation

$$E_t[\widetilde{C}^p_{x,t+1}\widetilde{R}'_{x,t+1}] = \frac{1}{2}F + \mathcal{O}(\epsilon^3)$$

- Rewritten with second-moment variables

$$\frac{\gamma}{\zeta}E_t[\eta\widetilde{Y}\widetilde{R}' + (\frac{\zeta}{\gamma} - \zeta - s)\widetilde{P}\widetilde{R}' - s\widetilde{I}\widetilde{R}' - \widetilde{D}\widetilde{R}' + \widetilde{Z}\widetilde{R}' + \alpha\widetilde{R}\widetilde{R}'] = \frac{1}{2}F$$

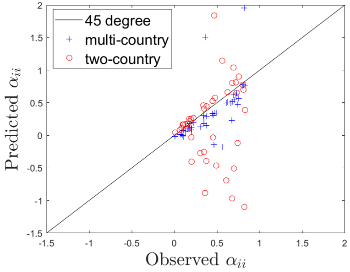
- No labor risk hedging — exclude labor income from wealth constraint

$$\frac{\gamma}{\zeta}E_t[\mu\eta\widetilde{Y}\widetilde{R}' + (\frac{\zeta}{\gamma} - \zeta - s)\widetilde{P}\widetilde{R}' - s\widetilde{I}\widetilde{R}' - \widetilde{D}\widetilde{R}' + \widetilde{Z}\widetilde{R}' + \alpha\widetilde{R}\widetilde{R}'] = \frac{1}{2}F$$

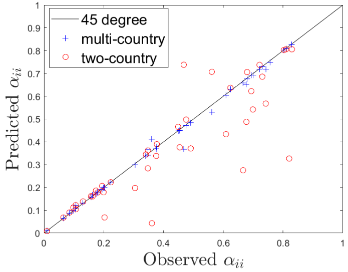
- No RER risk hedging — assume log utility ($\gamma = 1$)

$$\frac{\gamma}{\zeta}E_t[\eta\widetilde{Y}\widetilde{R}' - s\widetilde{P}\widetilde{R}' - s\widetilde{I}\widetilde{R}' - \widetilde{D}\widetilde{R}' + \widetilde{Z}\widetilde{R}' + \alpha\widetilde{R}\widetilde{R}'] = \frac{1}{2}F$$

Domestic Asset Holdings ($\bar{\alpha}_{ii}$) under 2- vs 43-countries



(a) No labor risk hedging



(b) No RER risk hedging

Finding: Risk hedging becomes less essential in explaining asset home bias given countries' multilateral linkages.

Bilateral Trade and Financial Linkages

- High correlation of bilateral linkages across two channels

$$Corr(\alpha_{ij}, \pi_{ij}) = 0.83, \quad \text{where } \pi_{ij} = \frac{\bar{\pi}_{ij} + \bar{\pi}_{ji}}{2}, \quad \alpha_{ij} = \frac{\bar{\alpha}_{ij} + \bar{\alpha}_{ji}}{2}, \quad \forall i, j \in [1, I]$$

- Two potential explanations (assuming given trade structure)
  - Trade influences second moments and therefore risk-sharing/-hedging patterns
  - Bi-trade and bi-financial linkages face common barriers such as policy/information frictions
- Counterfactual exercises based on the structural model

$Corr(\alpha_{ij}, \pi_{ij})$ data 0.83	No labor hedging	No RER hedging
	0.53	0.82
	No financial friction	Homogeneous financial friction
	0.17	-0.08

Finding: Synchronous bilateral financial and trade linkages are driven by the high correlation of frictions across the two channels.

Determinants of Bilateral Asset Positions

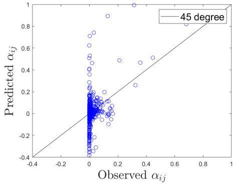
- Portfolio under no financial friction

$$\frac{\gamma}{\zeta}E_t[\eta\widetilde{Y}\widetilde{R}' + (\frac{\zeta}{\gamma} - \zeta - s)\widetilde{P}\widetilde{R}' - s\widetilde{I}\widetilde{R}' - \widetilde{D}\widetilde{R}' + \widetilde{Z}\widetilde{R}' + \alpha\widetilde{R}\widetilde{R}'] = 0$$

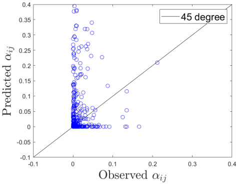
- Portfolio under homogeneous bilateral asset covariances

$$\frac{\gamma}{\zeta}E_t[\eta\widetilde{Y}\widetilde{R}' + (\frac{\zeta}{\gamma} - \zeta - s)\widetilde{P}\widetilde{R}' - s\widetilde{I}\widetilde{R}' - \widetilde{D}\widetilde{R}' + \widetilde{Z}\widetilde{R}' + \alpha\widetilde{R}\widetilde{R}'^{ctf}] = \frac{1}{2}F.$$

$$\textit{with} \quad \widetilde{R}\widetilde{R}'^{ctf}(i,j) = \bar{R}R_i, \quad \forall j \neq i \in \{1,2,...,I\text{-}1\}$$



(a) No Friction (complete markets)



(b) Homog. asset cov

Finding: Financial frictions and asset covariances relevant for risk diversification are important drivers for global financial allocation.

Empirical Evidence from Bilateral Asset Positions

Dep. Var: log(Bilateral Holdings)	(1)	(2)	(3)	(4)
log(GDP_o)	1.245 ***	1.108 ***	1.085 ***	1.111 ***
	(0.034)	(0.061)	(0.062)	(0.059)
log(GDP_d)	1.442 ***	-0.012	0.042	0.112
	(0.032)	(0.093)	(0.094)	(0.089)
log(dist)	-0.709 ***	-1.167 ***	-1.202 ***	-1.033 ***
	(0.037)	(0.021)	(0.022)	(0.022)
Chinn-Ito			0.674 **	2.288 ***
			(0.298)	(0.293)
corr(T)				5.049 ***
				(0.253)
Chinn × corr(T)				-4.501 ***
				(0.269)
Fixed Effects	N	Y	Y	Y
Gravity Var	N	Y	Y	Y
Observations	22,448	22,448	20,807	20,807
R^2	0.123	0.957	0.959	0.960

Robust standard errors in parentheses. Asset positions are from Factset/Lionshare. Fixed Effects include origin-, destination-, and time-FE. Gravity variables from CEPII. Chinn-Ito measures capital openness. corr(T) is estimated productivity correlation.

Presentation Outline

Joint Determination of Trade and Finance

- Two policy regimes: original or counterfactual $s \in \{org, ctf\}$
- Policy experiments with changes to trade or financial friction:

$$\hat{\tau}_{ij} = \frac{\tau_{ij}^{ctf}}{\tau_{ij}^{org}}, \qquad \widehat{F}(i,j) = \frac{F(i,j)^{ctf}}{F(i,j)^{org}}$$

- Two types of changes

$$\underbrace{\ln(Y_{i,t+1}^{ctf}) - \ln(Y_{i,t+1}^{org})}_{\text{Total changes}} = [\underbrace{\ln(\bar{Y}_i^{ctf}) - \ln(\bar{Y}_i^{org})}_{\text{Inter-regime changes}}] + \underbrace{[\ln(Y_{i,t+1}^{ctf}) - \ln(\bar{Y}_i^{ctf})] - [\ln(Y_{i,t+1}^{org}) - \ln(\bar{Y}_i^{org})]}_{\text{Intra-regime changes}}.$$

- Intra-regime analysis w/ linearization method to derive portfolios
- Inter-regime analysis w/ exact hat algebra technique

Inter-regime changes

- Exact hat algebra developed by DEK: $\widehat{A} = \frac{\bar{A}^{ctf}}{\bar{A}^{org}}$
- Variables to solve for: wage and price

$$\widehat{w'} = [\widehat{w}_1, \widehat{w}_2, ..., \widehat{w}_I], \qquad \widehat{P'} = [\widehat{P}_1, \widehat{P}_2, ..., \widehat{P}_I]$$

- Characterizing equations

$$\widehat{P}_i^{-\theta} = \sum_{j=1}^{I} \bar{\pi}_{ji}^{org} \widehat{\tau}_{ji}^{-\theta} (\widehat{w}_j^{(1-\mu)\eta} \widehat{P}_j^{\mu\eta+1-\eta})^{-\theta},$$

$$\widehat{w}_i \bar{Y}_i^{org} = \sum_{j=1}^{I} \frac{\bar{\pi}_{ij}^{org} \widehat{\tau}_{ij}^{-\theta} (\widehat{w}_i^{(1-\mu)\eta} \widehat{P}_i^{\mu\eta+1-\eta})^{-\theta}}{\sum_{k=1}^{I} \bar{\pi}_{kj}^{org} \widehat{\tau}_{kj}^{-\theta} (\widehat{w}_k^{(1-\mu)\eta} \widehat{P}_k^{\mu\eta+1-\eta})^{-\theta}} \widehat{w}_j \bar{Y}_j^{org} [1 - \bar{D}_j^{ctf} (1 - \frac{1}{\bar{\beta}})].$$

- wealth $\bar{D}^{ctf}$ solved from intra-regime changes for portfolios

Combined intra- and inter-regime changes

- Evaluate the portfolio equation within each regime

$$E_t[\widetilde{C}^p_{x,t+1}\widetilde{R}'_{x,t+1}]^s = \frac{1}{2}F^s,\ s \in \{org, ctf\}$$

- Take cross-regime difference to predict portfolio changes
- Add up bilateral holdings to obtain country-level wealth positions
- Use solved $\widehat{D}$ to update $\widehat{w}$, $\widehat{P}$ in the gravity system
- Update portfolios with cross-country covariances (2nd moments)
- Keep iterating until the solution to a joint fixed-point problem with $(\widehat{w}, \widehat{P}, \widehat{D})$ is obtained in general equilibrium. ▸ Algorithm ▸ Existence

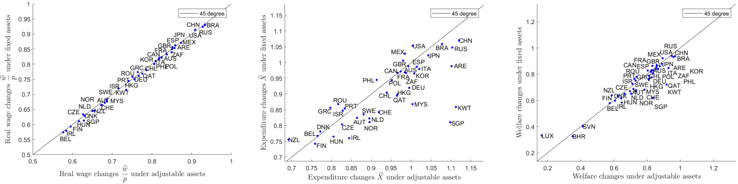
Policy Experiment 1: Universal Increase of Trade Cost by 20%

Welfare under fixed (y-axis) and adjustable (x-axis) asset positions

(a) Real Wage $\frac{\widehat{w}_i}{\widehat{p}_i}$

(b) Expenditure $\widehat{X}_i = \frac{1-\check{D}_i^{ctf}(1-\frac{1}{\beta})}{1-\check{D}_i^{org}(1-\frac{1}{\beta})}\widehat{Y}_i$

(c) Welfare $\widehat{\mathbb{W}}_i = \frac{\widehat{w}_i}{\widehat{P}_i}\frac{\widehat{X}_i}{\widehat{Y}_i}$



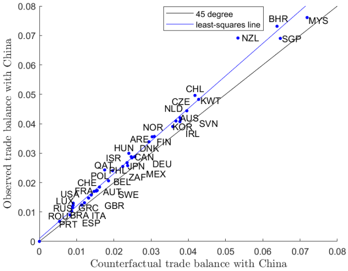
Finding: Many countries adjust asset positions and increase expenditure.

trade cost $\tau \Uparrow \rightarrow$ output sync $\tilde{Y}\tilde{Y}' \Downarrow \rightarrow$ asset cov $\tilde{R}\tilde{R}' \Downarrow \rightarrow$ wealth $\bar{D} \Uparrow$

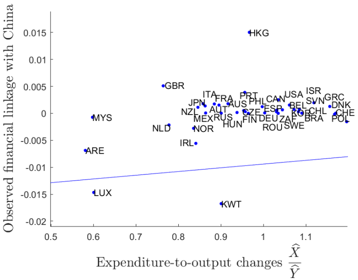
Households switch from trade to finance for international risk sharing.

Policy Experiment 2: Decrease of China's Financial Friction by 20%

(a) China's bi-trade surplus $(\bar{\pi}_{ij} - \bar{\pi}_{ji})^{org,ctf}$ with $i = \{CHN\}$



(b) Expenditure $\frac{\widehat{X}_i}{\widehat{Y}_i}$ and fin linkage $(\bar{\alpha}_{ij} + \bar{\alpha}_{ji})^{org}$ with $i = \{CHN\}$



Finding:

China's close partners benefit more from the country's financial liberalization.

Conclusion

This paper

- Combines portfolio choice solution method and hat algebra technique to solve a general equilibrium model with trade and finance
- Delivers policy implications with counterfactual analyses with trade costs and financial frictions

Future Applications in Open Economy Macro

- Transmission of monetary policy (Miranda-Agrippino and Rey 2020)
- Optimal currency invoicing for international trade (Gopinath et al 2020)
- Financial and trade adjustments during events like GFC and trade war (Gourinchas and Rey 2007, Caliendo and Parro 2022)

Presentation Outline

Country List

Name	Code	Name	Code	Name	Code	Name	Code
Australia	AUS	France	FRA	Luxembourg	LUX	Russia	RUS
Austria	AUT	Germany	DEU	Malaysia	MYS	Singapore	SGP
Bahrain	BHR	Greece	GRC	Mexico	MEX	Slovenia	SVN
Belgium	BEL	Hong Kong	HKG	Netherlands	NLD	Spain	ESP
Brazil	BRA	Hungary	HUN	New Zealand	NZL	Sweden	SWE
Canada	CAN	Ireland	IRL	Norway	NOR	Switzerland	CHE
Chile	CHL	Israel	ISR	Philippines	PHL	U.A.E.	ARE
China	CHN	Italy	ITA	Poland	POL	United Kingdom	GBR
Czech	CZE	Japan	JPN	Portugal	PRT	United States	USA
Denmark	DNK	Korea	KOR	Qatar	QAT	South Africa	ZAF
Finland	FIN	Kuwait	KWT	Romania	ROU		

Computation Strategy — System Reduction

Main Idea: Use the trade model to characterize intra-temporal allocation
Reduce the linear system to characterize inter-temporal allocation

- Forward-looking (Jump) variable S_1 and state variables S_2

$$S_1 = [C]', S_2 = [D, q, K, T]', S_{12} = [S_1, S_2]'$$

- Static (redundant) control variables and stochastic shocks S_4

$$S_3 = [P, w, r, Y, X, \pi, IV, L, R, d]'$$

- Original model

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} E_t \begin{bmatrix} \widetilde{S}_{12t+1} \\ \widetilde{S}_{3t+1} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \widetilde{S}_{12t} \\ \widetilde{S}_{3t} \end{bmatrix} + E_t(S_{4t+1})$$

- Use the gravity trade system to establish contemporaneous relationship

$$a_{21} = a_{22} = 0, \qquad \widetilde{S}_{3t} = -b_{22}^{-1} b_{21} \widetilde{S}_{12t} = G(\widetilde{S}_{12t})$$

- Reduced model

$$a_{11} E_t(\widetilde{S}_{12t+1}) + a_{12} G(E_t(\widetilde{S}_{12t+1})) = b_{11} \widetilde{S}_{12t} + b_{12} G(\widetilde{S}_{12t}) + E_t(S_{4t+1}).$$

- $14I + I^2 \Rightarrow 5I$ variables when performing eigen-decomposition of the DSGE model. Significantly improves efficiency and accuracy.

Back

Calibration

Comparative statics require few sufficient statistics which already incorporate trade and financial frictions

- 43 countries plus the rest of the world (ROW)
- GDP, NFA from Penn World Table (PWT)
- bilateral trade shares from DOTS, bilateral portfolio weights from Factset/Lionshare
- share of intermediate input in production $\eta = .312$ following DEK, share of labor input $1 - \mu$ as country-specific labor income share from PWT
- Ricardian productivity estimated following Levchenko and Zhang (2014) details
- trade elasticity $\theta = 4$ following Simonovska and Waugh (2014)

Back

Estimating Dynamic Productivity

Method developed by Levchenko and Zhang (2014)

- Theoretical prediction based on Eaton-Kortum model

$$\frac{\pi_{ij,t}}{\pi_{jj,t}} = \frac{T_{i,t}}{T_{j,t}} (\frac{\tau_{ij,t} c_{i,t}}{c_{j,t}})^{-\theta}.$$

- Regression equation

$$\ln(\frac{\pi_{ij,t}}{\pi_{jj,t}}) = \ln(T_{i,t}\hat{c}_{i,t}^{-\theta}) - \ln(T_{j,t}\hat{c}_{j,t}^{-\theta}) - \theta\hat{\tau}_{ij,t} + \gamma_{ij,t},$$

- Exponentiating importer FE

$$\widehat{Tc}_{j,t} = T_{j,t}\hat{c}_{j,t}^{-\theta}.$$

- Estimating production cost using PWT data

$$\hat{c}_{i,t} = (r_{i,t}^{\mu} w_{i,t}^{1-\mu})^{\eta} P_{i,t}^{1-\eta}.$$

- Recovering Ricardian productivity

$$T_{j,t} = T_{US,t}\frac{\widehat{Tc}_{j,t}}{\widehat{Tc}_{US,t}}(\frac{\hat{c}_{j,t}}{\hat{c}_{US,t}})^{\theta}$$

- Estimating dynamic productivity's ρ, $\bar{T}$ and Σ_T

Back

Algorithm

- Step 1. Calibrate the original steady state (s.s.) of the economy
- Step 2. Form initial guesses about inter-regime changes

$$(\widehat{w}^0, \widehat{P}^0, \widehat{D}^0)$$

- Step 3. Solve for portfolios around predicted counterfactual s.s.
- Step 4. Add up bilateral holdings to update country-level wealth

$$\check{D}_i^1 = \bar{\beta} \sum_{k=1}^{I-1} \check{\alpha}_{ik}^1 + \bar{\beta} \check{\alpha}_{iI}^1.$$

  with holdings of the numeraire asset following an updating rule

$$\check{\alpha}_{iI}^1 = \zeta_D^1 \check{\alpha}_{iI}^0 + (1 - \zeta_D^1)(\check{D}_i^0 / \bar{\beta} - \sum_{k=1}^{I-1} \check{\alpha}_{ik}^1),$$

  where ζ_D is obtained from asset I's market clearing condition

- Step 5. Use the solved $\widehat{D}^1$ to update inter-regime changes

$$\widehat{w}^1 = M(\widehat{w}^0) = \widehat{w}^0 (1 + \nu \frac{\mathbb{Z}_i(\widehat{w}^0)}{\bar{Y}_i^{org}})$$

- Step 6. Repeat steps 3-5 with updated variables until convergence where the joint fixed-point problem $(\widehat{w}, \widehat{P}, \widehat{D})$ is solved. Back

Existence and uniqueness of solution

Excess demand

$$\mathbb{Z}_i(\widehat{w}_i) = \frac{1}{\widehat{w}_i}[\widehat{w}_i \bar{Y}_i^{org} - \sum_{j=1}^{I} \frac{\bar{\pi}_{ij}^{org} \widehat{\tau}_{ij}^{-\theta} (\widehat{w}_i^{(1-\mu)\eta} \widehat{P}_i^{\mu\eta+1-\eta})^{-\theta}}{\sum_{k=1}^{I} \bar{\pi}_{kj}^{org} \widehat{\tau}_{kj}^{-\theta} (\widehat{w}_k^{(1-\mu)\eta} \widehat{P}_k^{\mu\eta+1-\eta})^{-\theta}} \widehat{w}_j \bar{Y}_j^{org} (1 - \check{D}_j^{ctf} (1 - \frac{1}{\bar{\beta}}))].$$

Properties of $\mathbb{Z}_i(w)$ by Alvarez and Lucas (2007)

- continuous
- homogenous of degree zero
- has the gross substitute property $\frac{\partial \mathbb{Z}_i(w)}{\partial w_j} > 0$
- satisfies Walras's Law ($\sum_i w_i \mathbb{Z}_i(w) = 0$)
- faces a lower but not upper bound
  $\mathbb{Z}_i(w) > -\max_j L_j, \max_i \mathbb{Z}_i(w \to w^{org}) \to \infty$

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