General Equilibrium Analysis of Multinational Financial and Trade Linkages

Chenyue Hu

University of California, Santa Cruz

December 21, 2023

Abstract

This paper develops a quantitative framework where trade and financial linkages are jointly determined in a multi-country setting. The model captures the potential interaction of finance and trade in general equilibrium: financial allocations reflect agents' risk-sharing patterns influenced by the global trade structure, and countries' asset positions shift demand in the world goods market. The model is solved with a novel approach that combines the linearization method for portfolio choice analysis in a DSGE framework and the exact hat algebra technique from the trade literature. The model yields important policy implications different from existing literature by characterizing the joint responses of finance and trade to frictions in the channels of globalization.

JEL Codes: F10, F30, F41, F44

Keywords: Portfolio choice in DSGE models, Multi-country RBC and Ricardian frameworks, Macro aspects of finance and trade

---

Email: chu78@ucsc.edu. I would like to thank Galina Hale, Alan Spearot, Eric van Wincoop, Nicolas Coeurdacier, Alan Taylor, Paul Bergin, Ina Simonovska, Katheryn Russ, Fabrizio Perri, Dave Donaldson, Kathryn Dominguez, Ken Kletzer, Hikaru Saijo, Ajay Shenoy, Justin Marion, Emile Marin, Barthlmy Bonadio, Maximiliano Dvorkin, Xian Jiang, and participants at the NBER IFM Spring Meeting, CEBRA IFM/ITM Annual Meeting, West Coast Trade Workshop, UC Santa Cruz and UC Davis seminars for comments and suggestions. Any mistakes are mine.

1 Introduction

Cross-country commodity and capital flows serve as two paramount engines of globalization. Nonetheless, few models have been proposed to characterize both trade and financial linkages in a multi-country structural framework. This paper develops a novel approach to build and solve a general equilibrium model with trade and financial channels. It has the potential to answer many unexplored questions about how the two channels interact, which provides new economic insights and policy implications. Examples of such questions include: how input-output linkages draw the map of global capital allocation, how a trade war reshapes countries' optimal asset positions, how regional financial integration influences the direction and volume of global trade flows, and how China's financial liberalization affects its bilateral trade ties with other economies.

The approach proposed in this paper not only combines the recent breakthroughs from both international macro and trade literatures, but also mitigates the methodological challenges faced by each strand and yields different predictions from existing works.[1] Compared to the trade literature that typically takes countries' asset positions as exogenous, this paper endogenizes global financial allocation under agents' intertemporal utility maximization decisions and bilateral financial frictions. Compared to the international macro literature that takes the real side of the economy as given when solving for portfolios, this paper captures the feedback effect of financial allocations on real variables.[2] Specifically, I embed portfolio choice analysis in a quantitative macro-trade model to examine trade and financial exchanges across 43 economies. The endogenous portfolios reflect agents' risk-sharing motives shaped by the global trade pattern, while countries' asset positions shift the world demand system in the goods market. Therefore, this approach permits a higher degree of interplay between the two channels of globalization, and facilitates a comprehensive understanding in the patterns and determinants of cross-country economic linkages.

[1]The two literatures have somewhat grown apart in recent decades. The international macro literature has made substantial progress in characterizing the fluctuations of macro fundamentals in response to stochastic shocks in a DSGE framework, but most analysis is conducted in a small open economy or two country model to deliver key mechanisms (for example, Mendoza (1991) and Backus et al. (1992)). Meanwhile, the trade literature has developed workhorse frameworks such as Eaton and Kortum (2002) and Melitz (2003) to efficiently characterize intra-temporal allocation across many economies. Most trade models, solved either in a static setting or under the assumption of predetermined shocks, cannot fully capture countries' risk characteristics essential for financial allocation. This paper merges the strengths of the two literatures to study the interaction of trade and finance in a multi-country setting.

[2]See a comprehensive survey of the international macro literature on the topic of portfolio choice and asset home bias by Coeurdacier and Rey (2013).

I apply the approach to a multi-country framework with Eaton and Kortum (2002)'s trade model embedded in an international real business cycle model with portfolio choice. Countries trade intermediate goods in the commodity market and equities as claims to capital income in the asset market. Financial frictions that vary across country pairs add costs to households' repatriation of foreign returns. Nevertheless, households have the incentive to build a diversified portfolio to reduce the impact of country-specific stochastic productivity shocks on their consumption for international risk sharing. To derive asset positions in incomplete markets, I follow the solution technique developed by Devereux and Sutherland (2011) and Tille and van Wincoop (2010), who combine a second-order linear approximation of the Euler equation with a first-order approximation of other equations to determine a steady-state portfolio. The technique is flexible enough to be applied to a wide range of DSGE models solvable with linearization methods.[3] These methods work well locally around a fixed steady state, but they do not predict counterfactual outcomes when economic conditions such as trade or financial policies shift the steady state of the economy.

I employ the 'exact hat algebra' technique, a nonlinear global solution method from the trade literature developed by Dekle et al. (2007), to overcome this challenge. This tractable technique, which requires few sufficient statistics for comparative statics analyses, efficiently characterizes linkages across many countries in a gravity system. In response to changes in cross-country trade or financial frictions, total changes of the economy include (1) the shift of steady states under different policy regimes (inter-regime changes), and (2) the behavior of variables under stochastic shocks around the steady state within a specific policy regime (intra-regime changes). I use exact hat algebra for (1) to measure the distance between steady states across original and counterfactual regimes globally, and the linearization method for (2) to derive equilibrium portfolios around the steady state of each regime locally. Countries' asset positions, determined by intra-regime changes of second-moment variables, will simultaneously influence inter-regime changes by shifting countries' expenditure levels. The counterfactual outcome is obtained as the solution to a joint fixed point problem of changes to wage on the real side and changes to portfolio on the financial side of the economy. By characterizing both the size and composition of countries' financial allocation, this approach makes an important contribution to the trade literature, which typically treats asset positions either as exogenously determined with values taken from the data or endogenously derived

[3] For example, Blanchard and Kahn (1980), Uhlig (1995), Sims (2002) among others.

under extreme circumstances like financial autarky or complete markets.[4] Moreover, the feedback of financial on real variables is missing from the macro literature, where portfolio is usually solved taking the real economy as fixed, without considering the two-way interactions of finance and trade, which jointly determine a steady state.

To illustrate the importance of jointly analyzing trade and financial channels, I conduct two experiments of policy changes in the two channels respectively. The first experiment examines a scenario where bilateral trade costs across countries uniformly rise by 20%. The counterfactual predictions from the model suggest that while real wage declines, many countries increase asset positions which allow them to raise expenditure. Households adjust their financial holdings because higher trade costs impair output synchronization, which also reduces cross-country asset covariances. The diversification benefits induce risk-averse households to hold more assets for international risk sharing. Therefore, adjustable asset positions in the financial channel mitigate lower real wage in the trade channel to leave welfare less affected by trade costs. This mechanism is not fully explored by the existing macro or trade literature, which may overestimate welfare loss due to the lack of an integrated general equilibrium framework.

The second experiment examines a scenario where China's bilateral financial frictions with others drop by 20% to reflect the country's recent efforts to improve its financial openness. The model predicts that a higher financial openness raises China's real wage by 2.5% and expenditure by 5.8%. This policy also generates heterogenous impacts on other countries, heavily influenced by their bilateral linkages with China. In the trade channel, the majority of countries increase exports to but decrease imports from China. The weakening of China's bilateral trade surplus is more pronounced for its closer trade partners. In the financial channel, countries with stronger existing financial linkages with China on average expect larger portfolio adjustments and expenditure increases. Therefore, welfare improvement is greater through both channels for countries including Korea and US, which share strong linkages with China. This quantitative model effectively captures policies' distributional impacts across many economies, with their bilateral linkages in trade and financial channels taken into full consideration.

This paper contributes to both international macro and trade literatures. The portfolio choice analysis employs Devereux and Sutherland (2011) and Tille and van Wincoop (2010)'s solution method, with a similar idea developed by Samuelson (1970) and Judd

[4] For example, Dekle et al. (2007) examine counterfactual trade patterns under financial autarky. Eaton et al. (2016) study the puzzles in international macroeconomics in complete markets. It is challenging to analyze general cases between these two extreme financial arrangements due to the difficulty of solving the portfolio choice problem in a multi-country general equilibrium model.

and Guu (2001) who propose a higher order approximation of the Euler equation to overcome the certainty equivalence of assets to a first order approximation of the model. This portfolio solution method has become a powerful tool in open economy macro to solve perplexing puzzles like asset home bias (for example, Coeurdacier and Rey (2013) and Coeurdacier and Gourinchas (2016)). Compared to alternative portfolio techniques driven by agents' specific preference for assets, including Pellegrino et al. (2021) and Liu et al. (2022), this method does not require separate demand assumptions for agents' intratemporal financial allocation. Portfolio is instead determined by endogenous second-moment variables, which reflect cross-country covariances of macro and financial variables under stochastic shocks of the economy in general equilibrium.[5] The asset home bias literature usually builds a two-country model in complete markets where analytical solutions are obtainable. This paper and Hu (2023a) acknowledge the existence of financial frictions and characterize global asset allocation across many uneven countries in incomplete markets. Furthermore, I combine the portfolio method with exact hat algebra for comparative statics policy analysis. Inter-regime analysis from this approach predicts the two-way interactions of trade and finance, while the international macro literature typically evaluates portfolios taking the trade pattern as given (such as Coeurdacier (2009), Heathcote and Perri (2013), Steinberg (2018), Chau (2020), and Hu (2020)).

This paper contributes to the trade literature by characterizing financial allocation in the Eaton and Kortum (2002) model embedded in a DSGE framework. Countries' asset positions shift demand in the world goods market, as shown by Dekle et al. (2007) who develop the hat algebra method originally proposed by Jones (1965) to examine counterfactual trade patterns without global imbalances. In this paper, both the size and composition of countries' asset positions are endogenously determined by households' inter- and intra-temporal decisions to maximize expected lifetime utility. Meanwhile, the world trade structure, which shapes cross-country covariances of macro variables under stochastic shocks, also influences global financial allocation. Last but not least, this paper adds to a growing literature that examines the interaction of trade and finance, including Antras and Caballero (2009), Fitzgerald (2012), Jin (2012), and Kehoe et al. (2018). I complement these works by developing techniques to characterize frictional financial and trade linkages in a multi-country structural model. This strand of literature is particularly valuable for understanding the patterns and determinants of globalization.

[5]See Hu (2023a) for a detailed diagnosis of how second-moment variables influence portfolio choice in a multi-country DSGE model. Mechanisms examined in the model include risk sharing, hedging, and diversification. These mechanisms are difficult to be fully captured by alternative portfolio techniques.

2 Model

This section develops a multi-country DSGE model with trade and financial linkages. Cross-country trade linkages are characterized by a Ricardian framework and financial linkages are characterized by households' portfolio choice driven by inter-temporal utility maximization decisions.

2.1 Production

There are I countries in the world indexed by $i \in \{1, 2, ..., I\}$. Each country i produces a final good with a continuum of intermediate goods u traded across countries

$$Q_{i,t} = \int_0^1 [q_{iu,t}(u)^{\frac{\epsilon-1}{\epsilon}} du]^{\frac{\epsilon}{\epsilon-1}}, \tag{1}$$

where ϵ is the elasticity of substitution in the CES aggregator. Country i's productivity for u at time t is the product of a good-specific component drawn from a time-invariant Fréchet distribution following the assumption of Eaton and Kortum (2002) (EK hereafter) and a country-specific productivity that affects all the varieties in i at t.

To characterize the risk of the world economy for portfolio analysis, I follow the international real business cycle literature to assume that country-level productivity $T_{i,t}$ follows an AR(1) process with mean $\bar{T}_i$ subject to stochastic shocks $\epsilon_{i,t}$ drawn from a joint normal distribution with a cross-country covariance matrix Σ_T:[6]

$$T_{i,t} = \rho T_{i,t-1} + (1-\rho)\bar{T}_i + \epsilon_{i,t}. \tag{2}$$

Production of intermediate goods combines labor L_i, capital K_i, and final goods. Let w_i, r_i, and P_i be the prices of these inputs, τ_{ij} be the iceberg trade cost for exports from country i to j. The share of i's goods in j's expenditure in this EK framework is

$$\pi_{ij,t} = \frac{T_{i,t}[\tau_{ij}(r_{i,t}^{\mu} w_{i,t}^{1-\mu})^{\eta} P_{i,t}^{1-\eta}]^{-\theta}}{\Phi_{j,t}}, \quad \text{with} \quad \Phi_{j,t} = \sum_{k=1}^{I} T_{k,t}[\tau_{kj}(r_{k,t}^{\mu} w_{k,t}^{1-\mu})^{\eta} P_{k,t}^{1-\eta}]^{-\theta}, \tag{3}$$

[6] This AR(1) process of country-specific productivity is a standard assumption from the international macro literature, such as workhorse DSGE frameworks by Mendoza (1991) with a small open economy and by Backus et al. (1992) with two symmetric countries. Besides productivity shocks, this model can be adapted to accommodate other risks that induce rational agents from different countries to construct portfolios for international risk sharing. Portfolio choice in such a DSGE model is shaped by endogenous cross-country covariances of macro variables under stochastic shocks in general equilibrium.

where $1-\eta$ is the share of final goods and $\frac{\mu}{1-\mu}$ is the capital-to-labor ratio in production. $\Phi_{j,t}$ determines the price of the final good in country j

$$P_{j,t} = \Gamma \Phi_{j,t}^{-\frac{1}{\theta}}, \tag{4}$$

in which Γ represents a Gamma function: $\Gamma(\frac{1-\epsilon}{\theta}+1)^{\frac{1}{1-\epsilon}}$.

Country i's intermediate goods market clearing condition follows

$$Y_{i,t} = \sum_{j=1}^{I} \pi_{ij,t} X_{j,t}, \tag{5}$$

where $Y_{i,t}$ is i's nominal output and $X_{j,t}$ is j's expenditure. The expenditure is used on consumption $C_{j,t}$, physical capital investment $IV_{j,t}$, or as intermediate input:

$$X_{j,t} = P_{j,t}(C_{j,t} + IV_{j,t}) + (1-\eta)Y_{j,t}. \tag{6}$$

2.2 Households

Intertemporal decisions on consumption and investment are made by a representative household in each country. The household supplies labor inelastically to earn wage income and makes forward-looking investment decisions to reduce the impact of country-specific productivity shocks on consumption in order to maximize expected lifetime utility

$$\max E_0 \sum_{t=0}^{\infty} \nu_t \frac{C_{i,t}^{1-\gamma}}{1-\gamma}. \tag{7}$$

γ is the coefficient of relative risk aversion for the CRRA utility and ν_t is the endogenous discount factor with a steady state value $\bar{\beta}$ which satisfies[7]

$$\nu_0 = 1, \qquad \nu_{t+1} = \nu_t \beta(C_{i,t}) \quad \text{with} \quad \beta(C_{i,t}) = \omega_i C_{i,t}^{-\psi}, \tag{8}$$

[7]This assumption follows Devereux and Sutherland (2011) who solve the portfolio choice problem with the local linearization method around a deterministic steady state in DSGE models. This endogenous discount factor is introduced to ensure a stationary wealth distribution in incomplete markets, otherwise even transitory shocks may have permanent impacts on wealth such that the steady state is indeterminate in a linearly approximated model (see Schmitt-Grohé and Uribe (2003) for a detailed discussion).

where $0 \leq \psi < \gamma$ and ω_i is a country-specific multiplier. The Euler equation for capital investment is derived from the household's utility maximization problem:

$$C_{i,t}^{\psi-\gamma} = \omega_i E_t[\frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}}((1-\delta)P_{i,t+1} + \frac{\partial Y_{i,t+1}}{\partial K_{i,t+1}})], \tag{9}$$

which yields the law of motion for capital accumulation subject to depreciation δ

$$K_{i,t+1} = (1-\delta)K_{i,t} + IV_{i,t}. \tag{10}$$

Besides physical capital, households invest in financial assets for consumption smoothing. I follow the asset home bias literature including Coeurdacier and Rey (2013) and Heathcote and Perri (2013) to assume that countries issue equities, which pay dividends as claims to capital income net of investment expenditure:[8]

$$d_{i,t} = \eta\mu Y_{i,t} - P_{i,t}IV_{i,t}. \tag{11}$$

Dividends $d_{i,t}$ and equity prices $q_{i,t}$ decide equity returns

$$R_{i,t+1} = \frac{d_{i,t+1} + q_{i,t+1}}{q_{i,t}}. \tag{12}$$

There exists bilateral financial friction across countries modeled as an iceberg transaction cost f_{ij} such that the household in country i expects to collect $e^{-f_{ij}}R_{j,t+1}$ when repatriating asset returns from country j.[9] Besides, these frictions are second-order in magnitude (proportional to the variance of shocks) to be consistent with the solution method for portfolio choice in a DSGE framework developed by Devereux and Sutherland (2011) and Tille and van Wincoop (2010). Acknowledging that assets are distinguishable

[8]This paper focuses on equities for which I have comprehensive data for bilateral holdings. Since equities are modeled as claims to capital income, they should in theory represent other forms of assets including bonds, bank loans, derivatives, reserves, and FDI. High-quality data that cover bilateral investment positions for all these assets are nonexistent to my knowledge. If such data become available, future theoretical frameworks can compare the patterns and determinants for different types of assets. A great example along this direction is Coeurdacier and Gourinchas (2016) who compare equity and bond risk-hedging positions in a two-country setting.

[9] See similar assumptions for financial friction in the international macro literature including Heathcote and Perri (2004) and Aviat and Coeurdacier (2007). Financial friction modeled as a transaction cost on asset returns may represent different barriers to global financial investment, including global financial liquidity, countries' capital account openness, and country pairs' geographic distance and bilateral financial agreements. It can take alternative forms such as informational frictions, as Okawa and van Wincoop (2012) show that these two types of frictions yield very similar predictions for portfolios.

by their risk characteristics, these authors combine a second-order approximation of the Euler equation

$$\frac{C_{i,t}^{\psi-\gamma}}{P_{i,t}} = \omega_i E_t[\frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}} R_{i,t+1}] = \omega_i E_t[\frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}} e^{-f_{ij}} R_{j,t+1}], \qquad \forall i, j \in \{1, ..., I\}. \tag{13}$$

with a first-order approximation of other equations to determine a zero-order (i.e. steady-state) portfolio. Portfolios derived from the Euler equation (13) capture both inter- and intra-temporal investment decisions of households to maximize their expected lifetime utility (7). Inter-temporally, households decide between financial investment and current consumption, given their discount factor β and elasticity of intertemporal substitution $\frac{1}{\gamma}$, upon expected asset returns $R_{j,t+1}$ and inflation $P_{i,t+1}$. Intra-temporally, the covariance matrix of countries' productivity shocks Σ_T and the matrix of bilateral financial frictions are reflected in the second-order Taylor expansion of the Euler equation, evaluating which determines portfolio choice. Therefore, households will naturally prefer assets from countries whose shocks are less correlated with their home country's for risk diversification, and whose assets are subject to lower frictions to maximize financial payoff.

Let $\lambda_{ik,t}$ be i's purchase of k's assets at the end of period t, and the supply of assets issued by any country k be normalized at unity $\sum_{i=1}^{I} \lambda_{ik,t} = 1$. Country i's net bilateral holdings defined as

$$\alpha_{ii,t} = q_{i,t}(\lambda_{ii,t} - 1), \qquad \alpha_{ij,t} = q_{j,t}\lambda_{ij,t}, \qquad \forall j \neq i, \tag{14}$$

will sum up to zero for the market clearing condition of asset k

$$\sum_{i=1}^{I} \alpha_{ik,t} = 0, \tag{15}$$

and to the net wealth position of holder country i denoted as $D_{i,t}$

$$D_{i,t} = \sum_{k=1}^{I} \alpha_{ik,t}. \tag{16}$$

Country i's wealth constraint hence follows

$$D_{i,t} = D_{i,t-1}e^{-f_{iI}} R_{I,t} + \sum_{k=1}^{I-1} \alpha_{ik,t-1}(e^{-f_{ik}} R_{k,t} - e^{-f_{iI}} R_{I,t}) + Y_{i,t} - X_{i,t}. \tag{17}$$

2.3 Portfolio Choice

Solving for equilibrium portfolios $\bar{\alpha}_{ik}$ with the method by Devereux and Sutherland (2011) and Tille and van Wincoop (2010) (DSTW hereafter) in a DSGE framework involves log-linearizing the model around the steady state of the economy. $\widetilde{A}_t$ denotes the log-deviation of any variable A from its steady state value $\bar{A}$ under stochastic shocks in the model

$$\widetilde{A}_t = \ln(\frac{A_t - \bar{A}}{\bar{A}}). \tag{18}$$

Countries' portfolios are derived from the second-order Taylor expansion of their Euler equations (13) around the steady state. Stacking the equations vertically with each row representing a holder country constructs a system of equations for the world bilateral portfolio matrix to be solved. These portfolio determination equations are summarized as (see Appendix B.2 for derivation):

$$E_t(\widetilde{C}^p_{x,t+1}\widetilde{R}'_{x,t+1}) = \frac{1}{2}F + \mathcal{O}(\epsilon^3), \tag{19}$$

$\widetilde{C}^p_{x,t+1}$ is the vector of countries' price- and utility-adjusted consumption ($C^p_{i,t+1} = \frac{P_{i,t+1}}{C^{-\gamma}_{i,t+1}}$) relative to the numeraire country I's. $\widetilde{R}_{x,t+1}$ is the vector of countries' asset returns in excess of I's. Therefore, $\widetilde{C}^p_{x,t+1}\widetilde{R}'_{x,t+1}$ represents the covariance matrix of countries' consumption differentials and excess returns, whose element in the i^{th} row j^{th} column represents the covariance between i's consumption differential and j's excess return. This covariance determines i's portfolio including its holding of j's asset, which is also influenced by bilateral financial friction f_{ij} embedded in the friction matrix F in Equation 19. $\mathcal{O}(\epsilon^3)$ captures all terms of order higher than two from the second-order Taylor expansion.

The solution to Equation 19 will determine the world portfolio matrix:[10]

$$\begin{bmatrix} \bar{\alpha}_{11} & \bar{\alpha}_{12} & \cdots & \bar{\alpha}_{1I\text{-}1} \\ \bar{\alpha}_{21} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \vdots & & & \bar{\alpha}_{I\text{-}2I\text{-}1} \\ \bar{\alpha}_{I\text{-}11} & \cdots & \bar{\alpha}_{I\text{-}1I\text{-}2} & \bar{\alpha}_{I\text{-}1I\text{-}1} \end{bmatrix} \tag{20}$$

[10]The dimension of the portfolio matrix determined by Equation 19 is (I-1) $\times$ (I-1) instead of $I \times I$. For the remaining assets positions, country i's holding of the numeraire asset from I is decided by the difference between i's aggregate asset position and its bilateral holding of non-numeraire assets. Meanwhile, numeraire country I's holding of any asset j is decided by j's market clearing condition, which ensures that the supply of the asset equals the demand.

whose element in the i^{th} row j^{th} column represents i's equilibrium holding of j's asset. Let $\check{\alpha}_{ij}$ be i's derived equilibrium holding adjusted for output and discount factor

$$\check{\alpha}_{ij} = \frac{1}{\bar{\beta}\bar{Y}_i}\bar{\alpha}_{ij}, \tag{21}$$

then i's equilibrium net wealth as shares of output is the sum of bilateral holdings

$$\check{D}_i = \frac{\bar{D}_i}{\bar{Y}_i} = \bar{\beta}\sum_{j=1}^{I}\check{\alpha}_{ij}. \tag{22}$$

Hu (2023a) examines mechanisms of portfolio choice behind Equation 19, which work through second-moment variables that reflect cross-country covariances of macro and financial variables in general equilibrium. From quantitative analysis, risk diversification influenced by the asset covariance structure and heterogeneity of bilateral financial frictions across countries turn out to be the major determinants of global financial allocation.

To conclude the model setup, the general equilibrium of the model consists of a set of prices and quantities such that 1) households decide on consumption and investment to maximize expected lifetime utility, 2) firms set output and price to maximize profit, and 3) factors, goods, and asset markets all clear.[11] The steady state of the economy is where stochastic shocks are turned off and endogenous variables are time invariant.

3 Computation

This section develops a computation strategy, which combines macro and trade methods, to examine the joint determination of financial and real variables in general equilibrium. In particular, I apply the strategy to conduct comparative statics analysis where there are policy changes to cross-country trade costs or financial frictions, in order to quantify the impacts of these barriers to globalization.

3.1 Strategy for Policy Analysis

I define two policy regimes denoted as $s \in \{\text{org,ctf}\}$ for original and counterfactual regimes respectively. The original regime of the model is calibrated to the data, while

[11] Goods and asset market clearing conditions are Equations 5 and 15. Factor market clearing ensures that wage and capital rental fee equalize the supply and demand for labor and capital respectively.

the counterfactual regime refers to the scenarios where trade costs (τ_{ij}) or financial frictions (f_{ij}) take alternative values in policy experiments. Either regime characterizes a steady state of the economy under specific trade costs and financial frictions. I quantify the changes to variables' steady-state values across the two regimes under different policies.[12] Such policy changes affect both the first moments (levels) and second moments (covariances) of variables, the latter of which determines countries' equilibrium portfolios.

Using country i's output at $t+1$ as an example, I decompose its total changes under counterfactual versus original frictions into two components

$$\underbrace{\ln(Y_{i,t+1}^{ctf}) - \ln(Y_{i,t+1}^{org})}_{\text{Total changes}} = \underbrace{[\ln(\bar{Y}_i^{ctf}) - \ln(\bar{Y}_i^{org})]}_{\text{Inter-regime changes}} + \underbrace{[\ln(Y_{i,t+1}^{ctf}) - \ln(\bar{Y}_i^{ctf})] - [\ln(Y_{i,t+1}^{org}) - \ln(\bar{Y}_i^{org})]}_{\text{Intra-regime changes}}. \tag{23}$$

From this decomposition, the total changes of a variable reflect 1) the change of its steady-state level under different policy regimes (inter-regime changes), and 2) the deviation of the variable from its steady state under stochastic shocks within a specific policy regime (intra-regime changes). Countries' equilibrium asset positions, determined by intra-regime changes of second-moment variables, will simultaneously influence inter-regime changes by shifting countries' equilibrium expenditure level in the goods market. Therefore, the counterfactual steady state of the economy is obtained as the solution to a joint fixed point problem of financial and real variables in general equilibrium.

I use 'exact hat algebra' developed by Dekle et al. (2007) (DEK hereafter), a global solution method from the trade literature, to characterize inter-regime changes. Let the ratio of any variable A's counterfactual to original steady-state value be denoted as

$$\widehat{A} = \frac{\bar{A}^{ctf}}{\bar{A}^{org}}. \tag{24}$$

The vectors of all the countries' wages and prices

$$\widehat{w}' = [\widehat{w}_1, \widehat{w}_2, ..., \widehat{w}_I], \qquad \widehat{P}' = [\widehat{P}_1, \widehat{P}_2, ..., \widehat{P}_I] \tag{25}$$

[12] This paper focuses on comparative statics analysis of steady states under policy changes rather than transition dynamics across steady states. This is because I use the local linearization method to solve for portfolio choice in DSGE models, where wealth as a state variable does not follow a stationary distribution in incomplete markets (Schmitt-Grohé and Uribe (2003)). Around a specific steady state, we can assume endogenous discount factors to induce stationarity under stochastic shocks following Devereux and Sutherland (2009), but this assumption does not apply to transition dynamics across steady states which shift globally.

are obtained by DEK's iterative computational procedure based on countries' price determination equation and goods market clearing condition, given counterfactual wealth $\check{D}^{ctf}$ and potential trade cost changes $\widehat{\tau}$:[13]

$$\widehat{P}_i^{-\theta} = \sum_{j=1}^{I} \bar{\pi}_{ji}^{org} \widehat{\tau}_{ji}^{-\theta} (\widehat{w}_j^{(1-\mu)\eta} \widehat{P}_j^{\mu\eta+1-\eta})^{-\theta}, \tag{26}$$

$$\widehat{w}_i \bar{Y}_i^{org} = \sum_{j=1}^{I} \frac{\bar{\pi}_{ij}^{org} \widehat{\tau}_{ij}^{-\theta} (\widehat{w}_i^{(1-\mu)\eta} \widehat{P}_i^{\mu\eta+1-\eta})^{-\theta}}{\sum_{k=1}^{I} \bar{\pi}_{kj}^{org} \widehat{\tau}_{kj}^{-\theta} (\widehat{w}_k^{(1-\mu)\eta} \widehat{P}_k^{\mu\eta+1-\eta})^{-\theta}} \widehat{w}_j \bar{Y}_j^{org} [1 - \check{D}_j^{ctf} (1 - \frac{1}{\bar{\beta}})]. \tag{27}$$

After solving for $\widehat{w}, \widehat{P}$ using the procedure, other real variables including $\widehat{Y}$ can be derived from equilibrium conditions in the model (see Appendix B.1 for details). Exact hat algebra is easy to implement as it only requires calibrating countries' initial output ($\bar{Y}_i^{org}$) and bilateral trade shares ($\bar{\pi}_{ij}^{org}$) in the original steady state. The inter-regime changes characterized by Equations 26-27 will predict the location of the counterfactual steady state around which intra-regime changes are characterized to update $\check{D}_j^{ctf}$.

To determine financial allocation within a regime, I use the local linearization method for DSGE frameworks together with the portfolio solution technique developed by DSTW around a steady state (see Appendix B.2 for details). In the original regime, the portfolio determination equation (19) can be written as

$$E_t[\widetilde{C}_{x,t+1}^p \widetilde{R}_{x,t+1}']^{org} = \frac{1}{2} F^{org} + \mathcal{O}(\epsilon^3). \tag{28}$$

where the second-moment variable $\widetilde{C}_{x,t+1}^p \widetilde{R}_{x,t+1}'$ covaries with countries' observed portfolios ($\check{\alpha}^{org}$), which are also influenced by the matrix of financial frictions F^{org}. Suppose the matrix becomes F^{ctf} under any changes to bilateral financial frictions, and the second-moment variable $\widetilde{C}_{x,t+1}^p \widetilde{R}_{x,t+1}'$ is re-computed in the loglinearized model around the new steady state under counterfactual frictions, then counterfactual portfolios $\check{\alpha}^{ctf}$ will satisfy

$$E_t[\widetilde{C}_{x,t+1}^p \widetilde{R}_{x,t+1}']^{ctf} = \frac{1}{2} F^{ctf} + \mathcal{O}(\epsilon^3). \tag{29}$$

The difference between 28 and 29 determines the shift of countries' equilibrium asset positions across regimes in response to changes in frictions.[14] Bilateral asset positions

[13] See Appendix B.1 for the derivation of Equations 26-27 from Equations 3-5. For policy analysis conducted in this paper, I assume that changes to bilateral trade costs or financial frictions are not substantial enough to shift the world steady-state asset return $\bar{R}$ whose inverse is the discount factor $\bar{\beta}$.

[14] These portfolio equations only pin down non-numeraire countries' (relative to I's) holdings. After

add up to country-level wealth positions (22), whose inter-regime changes are denoted as

$$\widehat{D}' = [\widehat{D}_1, \widehat{D}_2, ..., \widehat{D}_I], \quad \text{where} \quad \widehat{D}_i = \frac{\check{D}_i^{ctf}}{\check{D}_i^{org}}. \tag{30}$$

The derived wealth positions will shift demand in the gravity trade model ($\check{D}_j^{ctf}$ in 27) to update inter-regime changes of real variables.[15] The counterfactual steady state of the economy can therefore be characterized by the solution to a joint fixed point problem of $(\widehat{w}, \widehat{P}, \widehat{D})$ in general equilibrium. Appendix B.3 describes the algorithm to solve this fixed point problem.

3.2 Calibration

I calibrate the model to a world economy that consists of 43 countries (listed in Table A.1) plus the rest of the world (ROW) over the period of 2001-2021. This sample selection is heavily influenced by the availability of bilateral financial and trade data. I use data averaged over the sample period to calibrate the original steady state of the economy. Employing exact hat algebra to predict counterfactuals requires few sufficient statistics. On the real side of the economy, we need countries' GDP from the Penn World Table (PWT) and bilateral trade shares computed with the Comtrade data from the Centre for Prospective Studies and International Information (CEPII). Countries' expenditure on its own goods is calculated as the difference between its gross expenditure and total imports, both from the World Development Indicators (WDI) compiled by the World Bank (WB). On the financial side, I obtain countries' wealth position from the WB, which reports trade balance (TB_i) linked to wealth $\check{D}_i = \frac{\bar{D}_i}{\bar{Y}_i}$ through the wealth constraint (17) in the steady state

$$\check{D}_i(1 - \frac{1}{\beta}) = \frac{T\bar{B}_i}{\bar{Y}_i} = \frac{\bar{Y}_i - \bar{X}_i}{\bar{Y}_i}. \tag{31}$$

Bilateral portfolio weights are from Factset/Lionshare which covers institutional holding of equities for many countries.[16] These portfolio weights as well as trade shares are

following these equations to derive the relative holdings with the new second moments and financial frictions, I's holdings are solved to satisfy the market clearing condition of each asset (15).

[15]Since these real variables also determine financial variables including dividends and returns (11 and 12) as well as their second moments, portfolio choice both influences and is influenced by the covariance structure of asset returns in general equilibrium.

[16]See Hu (2023b) for details about the dataset and its consistency with macro datasets including the IMF's International Financial Statistics. The dataset has much better coverage than the Coordinated Portfolio Investment Survey (CPIS) especially for the bilateral equity positions of non-OECD countries.

sufficient statistics which already incorporate the influence of existing bilateral trade costs and bilateral financial frictions on trade patterns and asset allocations across all the country pairs. Therefore, we do not need to calibrate these frictions, which would take many efforts and additional assumptions, for comparative statics analysis.

Besides these sufficient statistics, parametrization of the model is also similar to that in Hu (2023a). For example, the risk of the economy is driven by productivity shocks. I follow Levchenko and Zhang (2014) to estimate countries' time-series Ricardian productivity consistent with the EK model (see Appendix B.5 for estimation details). Based on the estimated productivity, I obtain its persistence over time, country-specific time-averaged productivity, and cross-country covariance matrix. In addition, Table A.2 summarizes the values of other parameters, most of which are obtained from the macro or trade literature, for the calibration of this quantitative model.

4 Policy Analysis

This section exemplifies the application of the approach proposed in this paper with two policy experiments: a universal rise of bilateral trade costs and a reduction of China's financial frictions with other countries. These two policy experiments illustrate the interaction of trade and financial channels in general equilibrium, and deliver welfare implications by quantifying the impacts of barriers in these channels of globalization.

I consider both the relative cost ($\widehat{w}/\widehat{P}$) and size ($\widehat{X}/\widehat{Y}$) of expenditure when evaluating welfare. $\frac{\widehat{w}_i}{\widehat{P}_i}$ is the cross-regime change of i's equilibrium real wage, defined as the ratio of its nominal wage to price level, which reflects the purchasing power of labor income. $\frac{\widehat{X}_i}{\widehat{Y}_i}$ is the cross-regime change of i's equilibrium expenditure-to-output ratio, which is linked to the country's equilibrium wealth in each regime (Equation 31):

$$\frac{\widehat{X}_i}{\widehat{Y}_i} = \frac{1 - \check{D}_i^{ctf}(1 - \frac{1}{\beta})}{1 - \check{D}_i^{org}(1 - \frac{1}{\beta})}. \tag{32}$$

Welfare is therefore influenced by both real and financial variables in general equilibrium.

4.1 Universal Trade Cost Increase

I start policy analysis with a counterfactual scenario where bilateral trade costs among all the country pairs uniformly increase by 20%

$$\widehat{\tau}_{ij} = \frac{\tau_{ij}^{ctf}}{\tau_{ij}^{org}} = 1.2, \qquad \forall i \neq j \in \{1, ..., I\}. \tag{33}$$

This quantitative exercise, other than representing a worldwide tightening of trade policy including tariffs, can also reflect a situation where disruptions to global supply chains take place to make exchanges of goods more costly across countries.[17]

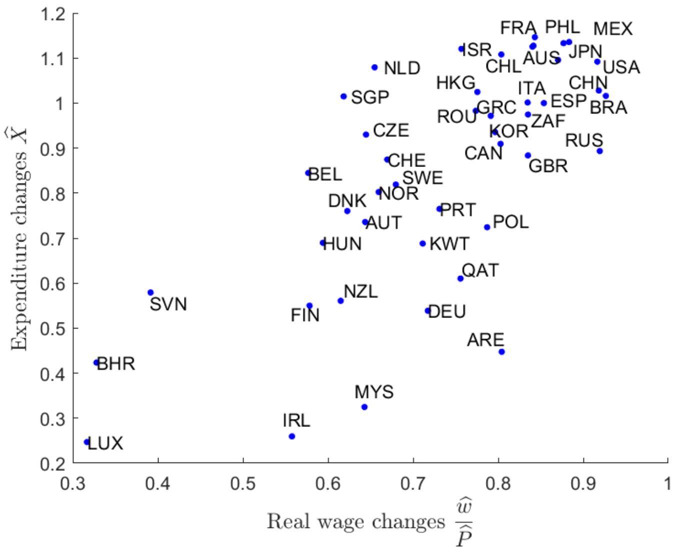
Figure 1 and Table A.3 report the impact of the trade cost increase on real wage. The median change in real wage across sample countries is $\widehat{w}/\widehat{P} = 0.76$, which implies that countries' purchasing power of labor income declines by 24% on average under the universal trade cost hike.[18] Larger countries, including both developed economies (for example, Japan and US) and emerging markets (China, Brazil, and Russia), suffer less real wage loss in general compared to small open economies heavily reliant on international trade (for example, Luxembourg and Slovenia). Figure 1 also explores the potential determinants of countries' real wage changes. The positive correlation between real wage and expenditure changes in 1a implies that, countries with greater increases in expenditure show smaller decreases in real wage. This result can be understood from the fact that countries' higher expenditure raises real wage by inducing a greater demand for labor, if much of the expenditure is spent on domestic goods. Therefore, the implication of expenditure for real wage is strong but not monotonic in 1a, because the composition of a country's expenditure also influences the ratio of its nominal wage to price level. Figure 1b plots the positive comovement between countries' real wage changes and their observed share of domestic goods in expenditure. Economies including Brazil, China, and US show stronger expenditure home bias with a higher $\bar{\pi}_{ii}^{org}$. Their increased expenditure mostly boosts their demand for domestic goods and therefore raises local wage compensation.

[17]This policy analysis assumes that iceberg trade costs do not generate revenues for a country distributable among its citizens. In addition, the magnitude of trade cost increases is assumed to be the same across country pairs to exclude any distributional effects caused by heterogeneous policy changes.

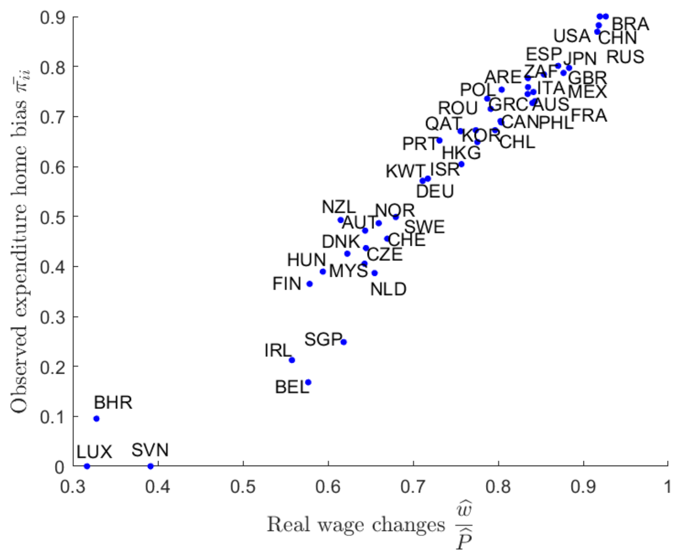
[18]For comparison, the original model and code by Dekle et al. (2007) would predict a median value 0.85 under the same counterfactual universal trade cost increase. The difference in the values is partly attributable to the fact that their model includes a non-tradable sector, which reduces the impact of trade costs on macro variables including wage. This paper focuses on a single tradable sector to deliver the main mechanism on the interaction between trade and finance. Future extensions of the model may consider multiple tradable and nontradable sectors as in Hu (2023b).

Figure 1: Real Wage under Trade Cost Increase

(a) Expenditure Change $\widehat{X}_i$

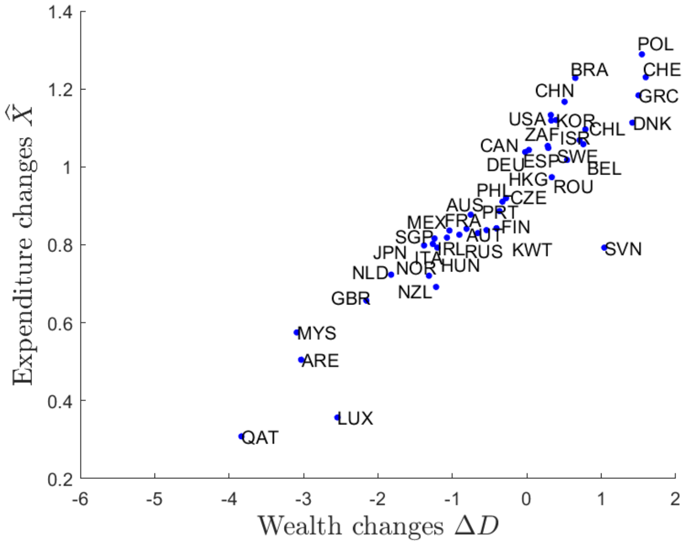


(b) Observed Expenditure Home Bias $\bar{\pi}_{ii}^{org}$



This figure presents real wage changes under a universal 20% bilateral trade cost increase. Countries' changes in real wage $\widehat{w}_i/\widehat{P}_i$ are on the horizontal axis, their cross-regime expenditure changes $\widehat{X}_i$ and observed shares of domestic goods in expenditure $\bar{\pi}_{ii}^{org}$ are on the vertical axis of 1a and 1b respectively.

Figure 2: Wealth and Expenditure Changes



This figure presents cross-regime changes under a universal 20% bilateral trade cost increase. Countries' changes in equilibrium wealth as shares of output $\Delta D_i = \check{D}_i^{ctf} - \check{D}_i^{org}$ are on the horizontal axis, their expenditure changes $\widehat{X}_i = \bar{X}_i^{ctf} / \bar{X}_i^{org}$ are on the vertical axis.

Expenditure changes depend on countries' financial adjustments, as shown in Figure 2 where countries with greater changes to equilibrium wealth position $\Delta D_i = \check{D}_i^{ctf} - \check{D}_i^{org}$ raise their equilibrium expenditure $(\widehat{X}_i)$ by a greater magnitude. Figure 3 further suggests that countries' financial reallocation is heavily influenced by their risk diversification pattern shaped by the asset covariance structure. Figure 3a plots the relation between country i's wealth change ΔD_i and the change to i's median asset covariance with others

$$\widehat{RR}_i = \frac{\bar{RR}_i^{ctf}}{\bar{RR}_i^{org}}, \text{ where } \bar{RR}_i^s = \text{median}[\widetilde{R}\widetilde{R}'^{ctf}(i,j)]^s, \ \forall j \in \{1, 2, ..., I\}, s \in \{org, ctf\}. \tag{34}$$

The negative correlation between ΔD_i and $\widehat{RR}_i$ implies that countries whose asset covariances with others decline tend to increase their country-level asset positions under the trade cost hike. This occurs since lower cross-country asset covariances yield more diversification benefits for these countries, which raise their asset holdings for international risk sharing. Figure 3b uses Germany as an example, and plots the changes of its bilateral holdings against its bilateral asset covariances with others. It suggests that Germany increases more of its holdings of assets from countries with which asset covariances drop, especially European economies including the Netherland and Belgium. These countries in close proximity share strong trade linkages with Germany. Under higher trade costs, their output synchronization greatly declines, which also lowers their asset covariances. Hence, Germany raises the weights of these assets in its portfolio for risk diversification. Such financial adjustments allow countries to raise expenditure that improves welfare.

Figure 4 combines countries' expenditure and real wage to conduct welfare analysis in two scenarios where countries have either fixed or adjustable asset positions. From Figure 4a, all the countries' real wage declines ($\widehat{w}/\widehat{P} < 1$) in both scenarios, because higher trade costs prohibit cross-country goods flows and raise the price level paid by households more than their nominal wage. The changes of real wage are quantitatively similar across the two scenarios, as countries are clustered around the 45 degree line. They exhibit a more disparate pattern in Figure 4b, where the majority of countries show greater expenditure when asset positions are adjustable than when they are fixed. In the latter case, countries' expenditure only moves at the same rate as their output ($\widehat{X} = \widehat{Y}$). In the former case, risk-averse households re-evaluate intertemporal decisions and adjust asset positions accordingly. When the trade channel, which facilitates output synchronization that generates higher asset covariances, faces greater barriers under trade costs, these countries increase asset positions to yield diversification benefits provided by the reduced asset covariances. Another angle to interpret this result is that, terms-of-trade movements in the trade channel, which would help reduce the impacts of idiosyncratic output shocks on consumption, are restricted by higher trade costs. Therefore, households switch from trade to financial channels for international risk sharing.

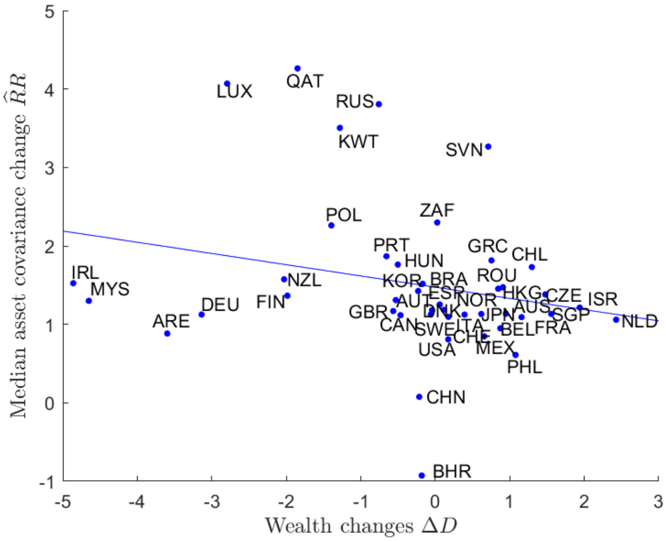
Due to adjustable asset positions which allow for expenditure expansion, more than half of the countries suffer less welfare loss under trade cost in Figure 4c, where welfare is measured as the product of equilibrium real wage and expenditure-to-output changes following DEK

$$\widehat{\mathbb{W}} = \frac{\widehat{w}}{\widehat{P}} \frac{\widehat{X}}{\widehat{Y}}. \tag{35}$$

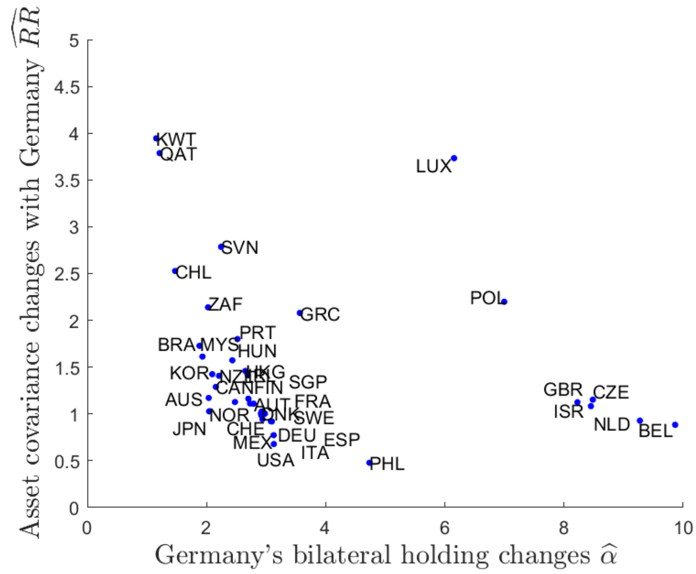
Hence, welfare consequences of the trade cost increase for these countries may be overestimated by standard trade models, which miss endogenous financial allocation as means for risk-averse households to smooth consumption intertemporally for utility maximization. Meanwhile, this feedback of countries' asset positions on trade patterns is not fully examined in the international macro literature, where portfolio is typically solved taking the real side of the economy's steady state as fixed. Therefore, this paper contributes to both literatures by capturing the two-way interactions of trade and finance.

Figure 3: Financial Allocation under Trade Cost Increase

(a) Wealth ΔD_i and Median Asset Covariance $\widehat{RR}_i$

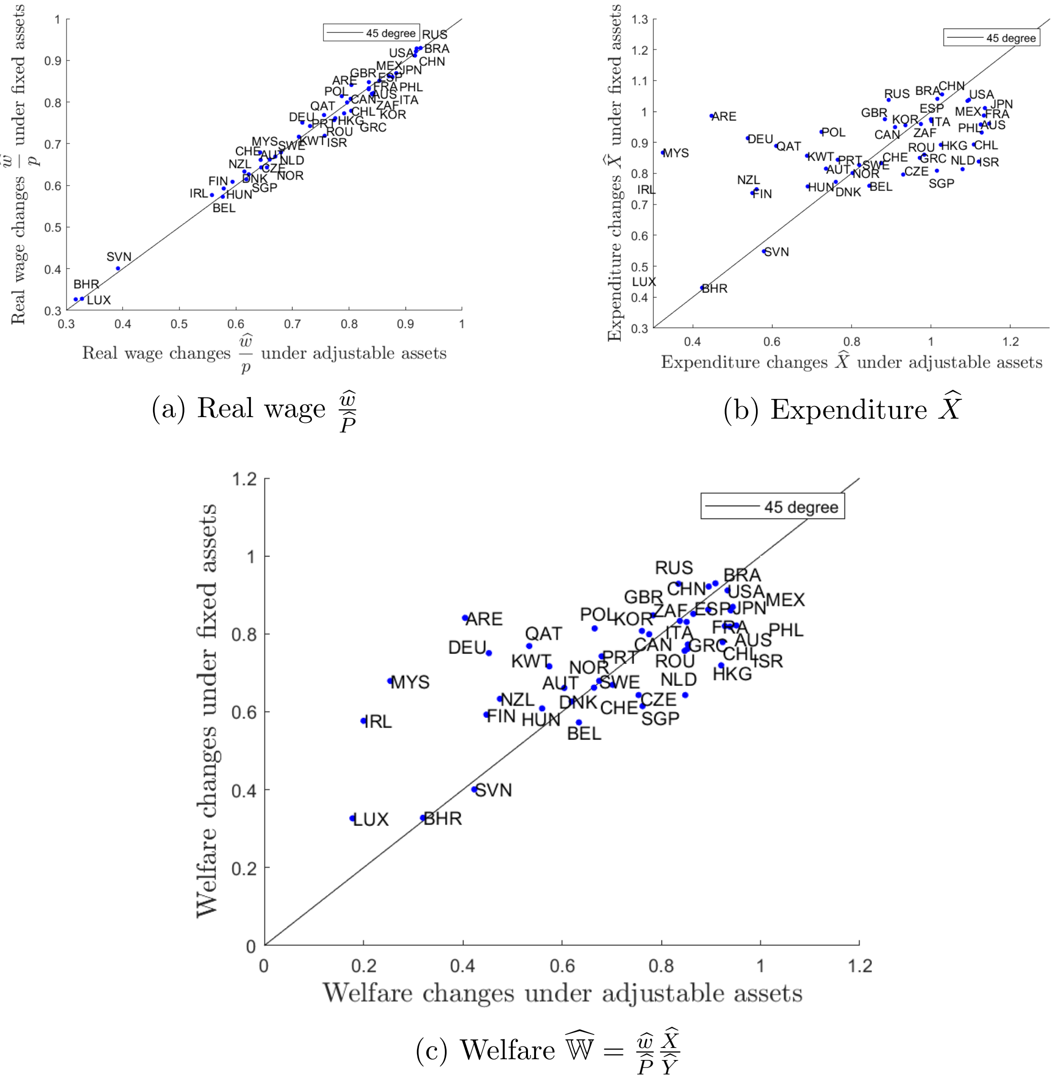


(b) Germany's Bilateral Holdings $\widehat{\alpha}_{DEU}$



This figure presents asset position changes under a universal 20% bilateral trade cost increase. Figure 3a plots the changes of countries' aggregate asset positions $\Delta D_i = \check{D}_i^{ctf} - \check{D}_i^{org}$ and their changes of median asset covariances with others $\widehat{RR}_i = \bar{RR}_i^{ctf} / \bar{RR}_i^{org}$. Figure 3b plots the changes of Germany's bilateral holdings $\widehat{\alpha}_{DEU} = \check{\alpha}_{DEU}^{ctf} / \check{\alpha}_{DEU}^{org}$ against the changes of its asset covariances with other countries $\widehat{RR}_{DEU} = RR_{DEU}^{ctf} / RR_{DEU}^{org}$.

Figure 4: Welfare with Fixed and Adjustable Asset Positions under Trade Cost Increase



(a) Real wage $\frac{\widehat{w}}{\widehat{P}}$

(b) Expenditure $\widehat{X}$

(c) Welfare $\widehat{\mathbb{W}} = \frac{\widehat{w}}{\widehat{P}}\frac{\widehat{X}}{\widehat{Y}}$

This figure compares welfare in two scenarios where countries have either fixed or adjustable asset positions under a universal 20% increase of bilateral trade costs. The horizontal (vertical) axes report variables in the scenario with adjustable (fixed) asset positions. Cross-regime changes are plotted for real wage $\widehat{w}/\widehat{P}$ in (a), expenditure $\widehat{X}$ in (b), and welfare $\widehat{\mathbb{W}}$ in (c).

4.2 China's Improved Financial Openness

The second policy analysis examines a scenario where China eases restrictions on inbound and outbound financial investment. In the past decade, China has pushed forward policies for financial liberalization, including adopting greater exchange rate flexibility, opening up asset markets for foreign investors, and expanding the use of RMB for international trade. These policies influence both China itself and other economies through bilateral economic linkages in financial and trade channels. It is meaningful to examine the welfare implications of China's improved financial openness.

To conduct the experiment, I assume China's bidirectional financial frictions with other countries decrease by 20%. These changes of frictions are in relative terms to the frictions faced by the numeraire country I, as the element in the i^{th} row j^{th} column of the friction matrix from the portfolio determination equation (19 or B.31) is

$$F(i,j) = (f_{iI} - f_{ij}) - (f_{II} - f_{Ij}). \tag{36}$$

Everything else equal, a decrease in bilateral friction f_{ij}, which stands for country i's friction when holding j's asset, alters the matrix of financial frictions by

$$\widehat{F}(i,j) = \frac{F(i,j)^{ctf}}{F(i,j)^{org}} > 1 \quad \text{with} \quad f_{ij}^{ctf} < f_{ij}^{org}. \tag{37}$$

Hence in our calibrated exercise, elements in the cross-regime change of the friction matrix are ones except for

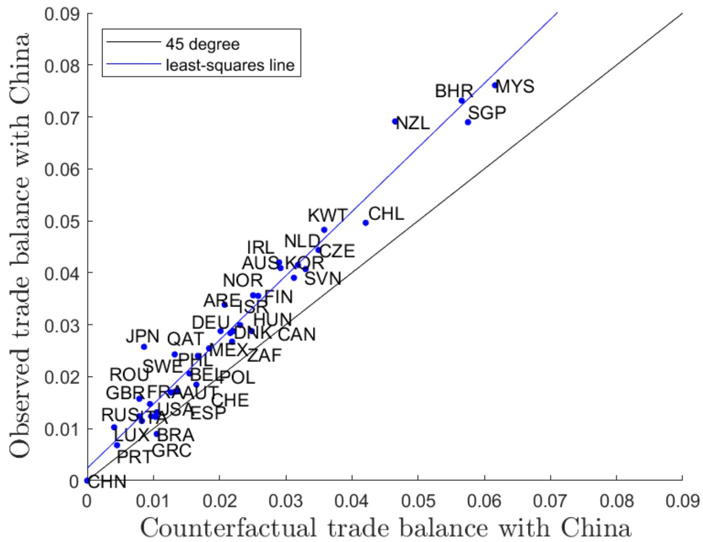
$$\widehat{F}(i,j) = \widehat{F}(j,i) = 1.2, \qquad i = \{CHN\}, \qquad \forall j \neq i \in \{1, ..., I\}. \tag{38}$$

As reported in Table A.4, the model predicts that a higher financial openness will improve China's equilibrium real wage by 2.5% and expenditure-to-output ratio by 5.8% under adjusted equilibrium asset positions. Furthermore, China's output grows by 10.3% due to financial liberalization. Given the country's importance in the global economy, the shift of China's policy generates modest and heterogeneous welfare impacts for other countries. I explore this cross-country heterogeneity in detail through an examination of both trade and financial channels.

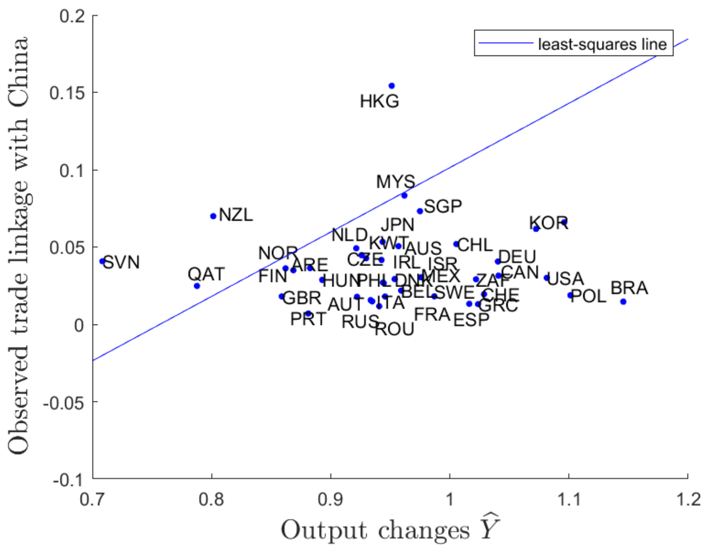
Figure 5 shows China's bilateral trade linkages with other countries under its reduced financial frictions. Figure 5a plots the difference between the share of China's goods in

Figure 5: Trade and Output under China's Higher Financial Openness

(a) Changes of Bilateral Trade Linkages with China



(b) Output changes and Observed Trade Linkages with China



This figure presents the impact of a 20% decrease in bilateral financial frictions between China and other countries in the trade channel. 5a plots the difference between countries' import share from and export share to China: $\bar{\pi}_{ij} - \bar{\pi}_{ji}, i = \{CHN\}, \forall j \in \{1, ..., I\}$. Values in original and counterfactual scenarios are depicted on the vertical and horizontal axis respectively. In 5b, countries' changes in output $\widehat{Y}$ are on the horizontal axis, their observed trade linkages with China $\bar{\pi}^{org}_{ij} + \bar{\pi}^{org}_{ji}$ in the original scenario are on the vertical axis. A 45 degree line and a linear fit line with least squares are labeled in the figure.

others' expenditure and the share of others' goods in China's expenditure:

$$\bar{\pi}_{ij} - \bar{\pi}_{ji}, \qquad i = \{CHN\}, \qquad \forall j \in \{1, ..., I\}. \tag{39}$$

Based on quantitative results predicted from the model, the majority of countries increase exports to China and decrease imports from China in the counterfactual scenario:

$$\bar{\pi}_{ij}^{ctf} < \bar{\pi}_{ij}^{org}, \qquad \bar{\pi}_{ji}^{ctf} > \bar{\pi}_{ji}^{org}, \qquad i = \{CHN\}, \tag{40}$$

which accounts for the pattern that they lie above the 45-degree line in Figure 5a. Financial liberalization hence weakens China's current account surplus. Furthermore, the slope of a linear fit line with least squares for the scatter plots is greater than 45 degrees. This finding suggests that the weakening of China's bilateral trade balance is stronger for its closer trade partners with large existing trade surplus such as Malaysia and Singapore. Moreover, China's output growth generates positive spillover effects for these economies, whose output also grows through tight trade linkages with China. Figure 5b plots countries' output changes against their observed trade linkages in the original regime calculated as the sum of bidirectional trade shares with China

$$\bar{\pi}_{ij}^{org} + \bar{\pi}_{ji}^{org}, \qquad i = \{CHN\}, \qquad \forall j \in \{1, ..., I\}. \tag{41}$$

The positive correlation between the two variables suggest that countries with stronger existing trade ties with China benefit more from its financial openness. Major trade partners of China including Malaysia, Singapore, Japan, and Korea are among the economies with the highest rises in output.

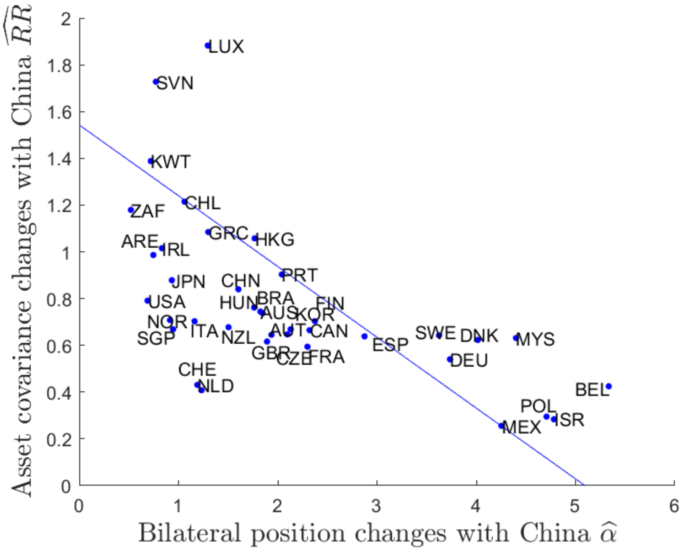
Figure 6 illustrates countries' asset reallocation under China's reduced financial frictions. In particular, I evaluate the change of China's bilateral financial linkages with other countries as the average change of their bidirectional asset positions

$$\frac{1}{2}(\widehat{\alpha}_{ij} + \widehat{\alpha}_{ji}), \qquad \text{where} \qquad \widehat{\alpha}_{ij} = \frac{\breve{\alpha}_{ij}^{ctf}}{\breve{\alpha}_{ij}^{org}}, \qquad i = \{CHN\}, \qquad \forall j \in \{1, ..., I\}, \tag{42}$$

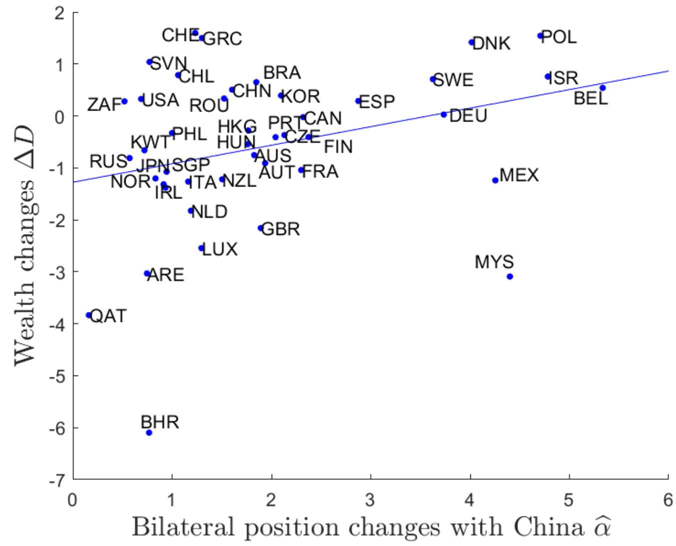
whose pattern is depicted in 6a. The figure shows that, despite the same magnitude of reduction in bilateral financial frictions, changes in China's financial linkages vary significantly across countries. Specifically, countries whose asset return covariance decreases with China's are more likely to strengthen bilateral financial linkages with the country, shown as the negative correlation between $\widehat{RR}$ and $\widehat{\alpha}$ in Figure 6a. Therefore, the asset

Figure 6: Financial Allocation under China's Higher Financial Openness

(a) Changes of Bilateral Financial Linkages with China



(b) Changes of Equilibrium Wealth



This figure presents the impact of a 20% decrease in bilateral financial frictions between China and other countries in the financial channel. Figure 6a plots the changes of China's bilateral financial linkages with other countries $\frac{1}{2}(\widehat{\alpha}_{CHN,j} + \widehat{\alpha}_{j,CHN}), \forall j \in \{1, ..., I\}$ against the changes of its asset covariances with others $\widehat{RR} = RR^{ctf}_{CHN,j}/RR^{org}_{CHN,j}$. Figure 6b plots countries' changes in aggregate wealth positions $\Delta D_i = \check{D}^{ctf}_i - \check{D}^{org}_i$ against their changes of bilateral financial linkages with China $\frac{1}{2}(\widehat{\alpha}_{CHN,j} + \widehat{\alpha}_{j,CHN})$.

covariance structure reshaped by the new global trade pattern under the policy change causes heterogeneous financial reallocation. The portfolio allocation at the bilateral level will decide countries' aggregate wealth position. Figure 6b shows that countries whose bilateral financial linkages with China strengthen are more likely to increase their equilibrium wealth. Hence, countries' size and composition of financial allocation both shift due to China's improved financial openness.

Figure 7 brings together trade and financial channels to evaluate the impact of China's financial liberation for countries' welfare, which considers their real wage and expenditure-to-output ratio. Figure 7a implies that countries with stronger existing trade linkages with China are predicted to experience larger real wage increases in general. Since these economies' output is higher as previously shown in Figure 5, their labor income adjusted for price level also tends to be higher. On the other hand, Figure 7b suggests that countries with stronger existing financial linkages with China on average expect larger expenditure-to-output increases. Therefore, both real wage in the trade channel and wealth in the financial channel rise more in favor of economies with stronger economic linkages with China. As a result, Korea, US, and Brazil are among the countries expected to benefit through both channels and experience large welfare improvements when China reduces financial frictions.

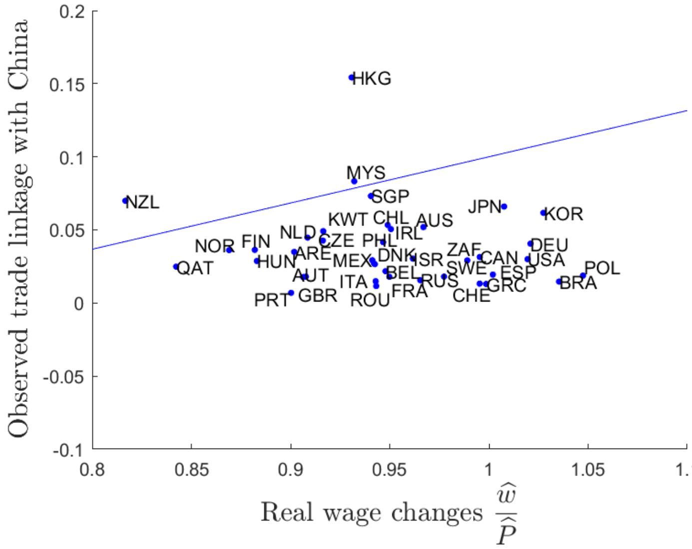
From the general equilibrium analysis above, China's financial liberalization generates heterogeneous welfare impacts on other economies, heavily influenced by their bilateral linkages in both trade and financial channels. The policy impacts of bilateral financial frictions on trade patterns have not been thoroughly investigated by the trade literature. Furthermore, the distributional impacts of policies examined in this multi-country setting are not fully captured by macro models with a small open economy or with two countries. Therefore, this unified multi-country framework with frictional trade and financial linkages has the potential for wide applications in open economy macro. For example, it can be employed to answer important questions including the transmission of monetary policy, currency invoicing of international trade, and simultaneous adjustments of international finance and trade during major global economic events.

5 Conclusion

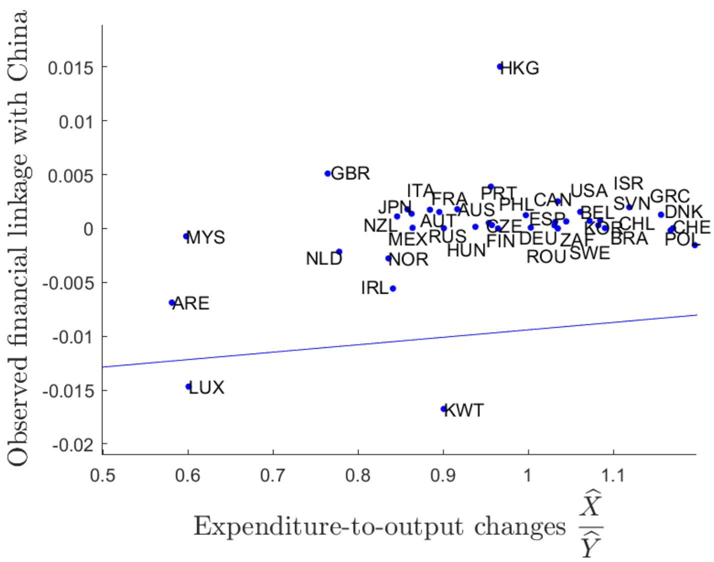
This paper develops a multi-country macro model where financial and trade channels interact with each other. The general equilibrium effects captured by the model provide new insights on the patterns and determinants of cross-country economic linkages. The

Figure 7: Real Wage and Expenditure under China's Higher Financial Openness

(a) Real Wage and Observed Trade Linkages with China



(b) Expenditure and Observed Financial Linkages with China



This figure presents the impact of a 20% decrease in bilateral financial frictions between China and other countries. Figure 7a plots countries' observed trade linkages with China $\bar{\pi}_{ij}^{org} + \bar{\pi}_{ji}^{org}$, $i = \{CHN\}, \forall j \neq i \in \{1, ..., I\}$ in the data against their changes of real wage $\widehat{w}/\widehat{P}$. Figure 7b plots countries' observed financial linkages with China $\breve{\alpha}_{ij}^{org} + \breve{\alpha}_{ji}^{org}$ against their changes of expenditure-to-output ratio $\widehat{X}/\widehat{Y}$.

solution to the model is derived with a novel approach that combines a local linear method for portfolio choice developed by DSTW from the international macro literature and a global nonlinear method 'exact hat algebra' developed by DEK from the international trade literature. This approach can readily be applied to a wide range of topics on how trade and finance influence each other, which have not been fully answered by either literature due to the lack of available quantitative framework. Meanwhile, the approach has many potentials for future extensions, among which I discuss one direction below.

This paper focuses on comparative statics analysis between steady states characterized by policy regimes, without tracing the dynamic path of the economy across steady states. Although the portfolio choice problem considers agents' intertemporal investment decisions, the derived equilibrium (steady-state) portfolio is static in nature. If future research questions involve time-series patterns of economic activities, solving dynamic portfolios requires extending the current method to higher-order approximations of the model. DSTW show that the first-order dynamics of portfolios are obtained by combining a third-order approximation of the portfolio determination equation with a second-order approximation of the rest of the model. However, this portfolio solution technique, like all the local linearization methods for DSGE models developed by Blanchard and Kahn (1980) and Uhlig (1995) among others, only works well around a steady state locally. When productivity shocks relevant for second moments of the economy are stochastic (instead of being deterministic as assumed by many trade models), we cannot use the method to trace dynamics of the economy under stochastic shocks across steady states. This problem is exacerbated by the fact that wealth as a state variable does not follow a stationary distribution in incomplete markets even locally (Schmitt-Grohé and Uribe (2003)). Around a steady state, we can introduce endogenous discount factors or life-cycle elements following DSTW to induce stationarity, but these assumptions do not apply to the transition dynamics across steady states which shift globally. To overcome this challenge, we may need to employ global solution methods (such as value or policy function iterations) to solve the portfolio choice problem in incomplete markets. To our knowledge, the existing global methods are not powerful enough to accommodate a large state space constituted by many countries (also known as the curse of dimensionality). But if future global solution methods with high efficiency and tractability become available, they may predict the full transition paths of financial and real variables under policy changes. Such dynamic analyses characterize the pattern and speed of convergence towards a steady state, which are important in quantifying both persistent and transitory economic outcomes when general equilibrium of the world economy is being restored.

References

Antras, P. and Caballero, R. J. Trade and capital flows: A financial frictions perspective. *Journal of Political Economy*, 117(4):701–744, 2009.

Aviat, A. and Coeurdacier, N. The geography of trade in goods and asset holdings. *Journal of International Economics*, 71(1):22–51, 2007.

Backus, D. K., Kehoe, P. J., and Kydland, F. E. International real business cycles. *Journal of Political Economy*, 100(4):745–775, 1992.

Blanchard, O. J. and Kahn, C. M. The solution of linear difference models under rational expectations. *Econometrica*, pages 1305–1311, 1980.

Chau, V. International portfolio investments with trade networks. *Available at SSRN 3740993*, 2020.

Coeurdacier, N. Do trade costs in goods market lead to home bias in equities? *Journal of International Economics*, 77(1):86–100, 2009.

Coeurdacier, N. and Gourinchas, P.-O. When bonds matter: Home bias in goods and assets. *Journal of Monetary Economics*, 82:119–137, 2016.

Coeurdacier, N. and Rey, H. Home bias in open economy financial macroeconomics. *Journal of Economic Literature*, 51(1):63–115, 2013.

Dekle, R., Eaton, J., and Kortum, S. Unbalanced trade. *American Economic Review: Papers and Proceedings*, 97(2):351–355, 2007.

Devereux, M. B. and Sutherland, A. A portfolio model of capital flows to emerging markets. *Journal of Development Economics*, 89(2):181–193, 2009.

Devereux, M. B. and Sutherland, A. Country portfolios in open economy macro models. *Journal of the European Economic Association*, 9(2):337–369, 2011.

Eaton, J. and Kortum, S. Technology, geography, and trade. *Econometrica*, 70(5):1741–1779, 2002.

Eaton, J., Kortum, S., and Neiman, B. Obstfeld and rogoff's international macro puzzles: a quantitative assessment. *Journal of Economic Dynamics and Control*, 72:5–23, 2016.

Fitzgerald, D. Trade costs, asset market frictions, and risk sharing. *American Economic Review*, 102(6):2700–2733, 2012.

Heathcote, J. and Perri, F. Financial globalization and real regionalization. *Journal of Economic Theory*, 119(1):207–243, 2004.

Heathcote, J. and Perri, F. The international diversification puzzle is not as bad as you think. *Journal of Political Economy*, 121(6):1108–1159, 2013.

Hu, C. Industrial specialization matters: A new angle on equity home bias. *Journal of International Economics*, 126:103354, 2020.

Hu, C. Portfolio choice analysis in a multi-country macro model, 2023a.

Hu, C. What explains equity home bias? theory and evidence at the sector level. *European Economic Review*, page 104585, 2023b.

Jin, K. Industrial structure and capital flows. *American Economic Review*, 102(5):2111–46, 2012.

Jones, R. W. The structure of simple general equilibrium models. *Journal of Political Economy*, 73(6):557–572, 1965.

Judd, K. L. and Guu, S.-M. Asymptotic methods for asset market equilibrium analysis. *Economic Theory*, 18(1):127–157, 2001.

Kehoe, T. J., Ruhl, K. J., and Steinberg, J. B. Global imbalances and structural change in the united states. *Journal of Political Economy*, 126(2):761–796, 2018.

Levchenko, A. A. and Zhang, J. Ricardian productivity differences and the gains from trade. *European Economic Review*, 65:45–65, 2014.

Liu, E., Redding, S., and Yogo, M. Goods trade and capital investments in the global economy. Technical report, 2022.

Melitz, M. J. The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, 71(6):1695–1725, 2003.

Mendoza, E. G. Real business cycles in a small open economy. *American Economic Review*, pages 797–818, 1991.

Okawa, Y. and van Wincoop, E. Gravity in international finance. *Journal of International Economics*, 87(2):205–215, 2012.

Pellegrino, B., Spolaore, E., and Wacziarg, R. Barriers to global capital allocation. Technical report, National Bureau of Economic Research, 2021.

Samuelson, P. A. The fundamental approximation theorem of portfolio analysis in terms of means, variances and higher moments. *The Review of Economic Studies*, 37(4): 537–542, 1970.

Schmitt-Grohé, S. and Uribe, M. Closing small open economy models. *Journal of International Economics*, 61(1):163–185, 2003.

Simonovska, I. and Waugh, M. E. The elasticity of trade: Estimates and evidence. *Journal of International Economics*, 92(1):34–50, 2014.

Sims, C. A. Solving linear rational expectations models. *Computational Economics*, 20 (1-2):1, 2002.

Steinberg, J. B. International portfolio diversification and the structure of global production. *Review of Economic Dynamics*, 29:195–215, 2018.

Tille, C. and van Wincoop, E. International capital flows. *Journal of International Economics*, 80(2):157–175, 2010.

Uhlig, H. A toolkit for analyzing nonlinear dynamic stochastic models easily. 1995.

Appendices

A Tables

Table A.1: List of Sample Countries

Name	Code	Name	Code	Name	Code	Name	Code
Australia	AUS	France	FRA	Luxembourg	LUX	Russia	RUS
Austria	AUT	Germany	DEU	Malaysia	MYS	Singapore	SGP
Bahrain	BHR	Greece	GRC	Mexico	MEX	Slovenia	SVN
Belgium	BEL	Hong Kong	HKG	Netherlands	NLD	Spain	ESP
Brazil	BRA	Hungary	HUN	New Zealand	NZL	Sweden	SWE
Canada	CAN	Ireland	IRL	Norway	NOR	Switzerland	CHE
Chile	CHL	Israel	ISR	Philippines	PHL	U.A.E.	ARE
China	CHN	Italy	ITA	Poland	POL	United Kingdom	GBR
Czech	CZE	Japan	JPN	Portugal	PRT	United States	USA
Denmark	DNK	Korea	KOR	Qatar	QAT	South Africa	ZAF
Finland	FIN	Kuwait	KWT	Romania	ROU		

This table lists the sample of countries included quantitative exercises. The sample selection is heavily influenced by the availability of bilateral financial and trade data over the sample period (2001-2021). All the other countries in the world as a whole are counted as the rest of the world (ROW).

Table A.2: Parametrization

Parameter	Description	Value	Source
θ	Trade Elasticity	4	Simonovska and Waugh (2014)
ψ	Elasticity of discount factor	0.01	Devereux and Sutherland (2009)
η	Share of intermediate input	0.312	Dekle et al. (2007)
1-μ	Labor share	Country-specific	Penn World Table
γ	Coefficient of relative risk aversion	2	Macro literature
$\bar{\beta}$	Annual discount factor	0.9	Macro literature
κ	Inverse of the Frisch elasticity	2	Macro literature
δ	Capital Depreciation	0.1	Macro literature

This table summarizes the parameter values for the calibration of the quantitative model. Most of the parameterization is directly taken from macro or trade literature.

B Computation

This section covers the two computation methods employed to solve the quantitative model. Appendix B.1 describes the 'exact hat algebra' technique from the international trade literature to characterize inter-regime changes globally. Appendix B.2 describes the linearization method for portfolio choice from the international macro literature to characterize intra-regime changes locally. Both inter- and intra-regime analyses are conducted to solve the joint fixed point problem of steady-state real and financial variables in

Table A.3: Welfare Analysis under Higher Trade Costs

Country	(I). Under Adjustable Asset Positions					(II). Under Fixed Asset Positions				
	$\widehat{w}=\widehat{Y}$	$\widehat{w}/\widehat{P}$	$\widehat{X}$	$\widehat{X}/\widehat{Y}$	$\widehat{\mathbb{W}}$	$\widehat{w}=\widehat{Y}$	$\widehat{w}/\widehat{P}$	$\widehat{X}$	$\widehat{X}/\widehat{Y}$	$\widehat{\mathbb{W}}$
AUS	1.019	0.840	1.125	1.104	0.928	0.958	0.820	0.958	1.000	0.820
AUT	0.783	0.644	0.736	0.939	0.604	0.816	0.661	0.816	1.000	0.661
BHR	0.434	0.328	0.424	0.976	0.320	0.431	0.328	0.431	1.000	0.328
BEL	0.768	0.577	0.845	1.099	0.634	0.760	0.573	0.760	1.000	0.573
BRA	1.035	0.927	1.016	0.981	0.909	1.041	0.930	1.041	1.000	0.930
CAN	0.959	0.803	0.909	0.948	0.761	0.950	0.808	0.950	1.000	0.808
CHL	0.964	0.804	1.108	1.149	0.924	0.893	0.779	0.893	1.000	0.779
CHN	1.054	0.918	1.028	0.975	0.896	1.055	0.922	1.055	1.000	0.922
CZE	0.795	0.645	0.930	1.170	0.754	0.796	0.643	0.796	1.000	0.643
DNK	0.764	0.623	0.760	0.995	0.620	0.773	0.627	0.773	1.000	0.627
FIN	0.710	0.578	0.550	0.774	0.448	0.737	0.593	0.737	1.000	0.593
FRA	1.016	0.843	1.146	1.128	0.951	0.961	0.822	0.961	1.000	0.822
DEU	0.854	0.717	0.539	0.631	0.453	0.914	0.751	0.914	1.000	0.751
GRC	0.901	0.791	0.972	1.079	0.854	0.850	0.773	0.850	1.000	0.773
HKG	0.932	0.776	1.025	1.099	0.853	0.892	0.761	0.892	1.000	0.761
HUN	0.731	0.594	0.689	0.943	0.560	0.758	0.609	0.758	1.000	0.609
IRL	0.724	0.558	0.260	0.359	0.200	0.750	0.577	0.750	1.000	0.577
ISR	0.921	0.757	1.120	1.217	0.921	0.838	0.719	0.838	1.000	0.719
ITA	0.982	0.835	1.001	1.020	0.851	0.969	0.831	0.969	1.000	0.831
JPN	1.063	0.883	1.136	1.069	0.944	1.011	0.869	1.011	1.000	0.869
KOR	0.961	0.796	0.936	0.974	0.776	0.956	0.799	0.956	1.000	0.799
KWT	0.852	0.711	0.688	0.808	0.574	0.857	0.717	0.857	1.000	0.717
LUX	0.440	0.317	0.247	0.562	0.178	0.452	0.327	0.452	1.000	0.327
MYS	0.822	0.643	0.325	0.395	0.254	0.867	0.679	0.867	1.000	0.679
MEX	1.057	0.877	1.133	1.072	0.940	0.987	0.860	0.987	1.000	0.860
NLD	0.833	0.655	1.079	1.295	0.848	0.814	0.643	0.814	1.000	0.643
NZL	0.726	0.615	0.561	0.772	0.475	0.749	0.633	0.749	1.000	0.633
NOR	0.797	0.660	0.802	1.007	0.665	0.801	0.662	0.801	1.000	0.662
PHL	1.011	0.841	1.127	1.115	0.939	0.932	0.818	0.932	1.000	0.818
POL	0.856	0.787	0.724	0.846	0.666	0.934	0.814	0.934	1.000	0.814
PRT	0.822	0.731	0.765	0.930	0.680	0.844	0.742	0.844	1.000	0.742
QAT	0.864	0.756	0.610	0.706	0.534	0.889	0.769	0.889	1.000	0.769
ROU	0.898	0.774	0.983	1.095	0.847	0.860	0.757	0.860	1.000	0.757
RUS	0.984	0.919	0.893	0.908	0.834	1.037	0.929	1.037	1.000	0.929
SGP	0.823	0.618	1.015	1.233	0.762	0.809	0.615	0.809	1.000	0.615
SVN	0.536	0.391	0.579	1.081	0.423	0.548	0.401	0.548	1.000	0.401
ESP	0.987	0.854	1.000	1.013	0.865	0.975	0.851	0.975	1.000	0.851
SWE	0.825	0.680	0.819	0.993	0.675	0.827	0.679	0.827	1.000	0.679
CHE	0.834	0.670	0.875	1.048	0.702	0.833	0.669	0.833	1.000	0.669
ARE	0.889	0.804	0.447	0.503	0.405	0.986	0.841	0.986	1.000	0.841
GBR	0.942	0.835	0.884	0.938	0.784	0.975	0.848	0.975	1.000	0.848
USA	1.072	0.917	1.092	1.019	0.934	1.034	0.912	1.034	1.000	0.912
ZAF	0.972	0.835	0.974	1.003	0.837	0.960	0.833	0.960	1.000	0.833
median	0.864	0.757	0.893	0.995	0.761	0.867	0.757	0.867	1.000	0.757
mean	0.865	0.724	0.834	0.953	0.697	0.862	0.725	0.862	1.000	0.725
std dev	0.150	0.148	0.254	0.211	0.219	0.140	0.145	0.140	0.000	0.145

This table present the welfare impact of a universal 20% bilateral trade cost increase for countries in the sample. Panels (I) and (II) report model predictions from scenarios where countries have adjustable and fixed asset positions respectively. Variables reported include cross-regime changes to countries' nominal wage ($\widehat{X}$, which equals output $\widehat{Y}$ based on Equation B.3), real wage ($\widehat{w}/\widehat{P}$), expenditure ($\widehat{X}$), expenditure-to-output ratio ($\widehat{X}/\widehat{Y}$), and welfare ($\widehat{\mathbb{W}} = \frac{\widehat{w}}{\widehat{P}}\frac{\widehat{X}}{\widehat{Y}}$). A hat value higher (lower) than one means the variable's steady-state value in the counterfactual scenario under the policy change is greater (smaller) than in the original scenario observed in the data.

Table A.4: Welfare Analysis under China's Financial Openness

Country	Nominal Wage $\widehat{w} = \widehat{Y}$	Real Wage $\widehat{w}/\widehat{P}$	Expenditure $\widehat{X}$	Expenditure-to-Output $\widehat{X}/\widehat{Y}$
AUS	0.957	0.951	0.877	0.917
AUT	0.922	0.906	0.826	0.896
BHR	0.503	0.496	0.095	0.190
BEL	0.959	0.948	1.018	1.061
BRA	1.145	1.035	1.228	1.072
CAN	1.041	0.995	1.038	0.997
CHL	1.005	0.967	1.096	1.090
CHN	1.103	1.025	1.167	1.058
CZE	0.926	0.909	0.887	0.957
DNK	0.954	0.941	1.113	1.167
FIN	0.883	0.882	0.842	0.954
FRA	0.946	0.950	0.837	0.885
DEU	1.040	1.021	1.043	1.003
GRC	1.024	0.999	1.184	1.156
HKG	0.951	0.931	0.920	0.967
HUN	0.893	0.883	0.838	0.938
IRL	0.943	0.947	0.793	0.841
ISR	0.976	0.962	1.058	1.085
ITA	0.935	0.943	0.802	0.858
JPN	0.944	0.949	0.798	0.846
KOR	1.072	1.027	1.120	1.045
KWT	0.921	0.916	0.830	0.901
LUX	0.594	0.580	0.357	0.601
MYS	0.962	0.932	0.575	0.598
MEX	0.945	0.942	0.816	0.864
NLD	0.930	0.916	0.723	0.778
NZL	0.801	0.817	0.692	0.863
NOR	0.862	0.869	0.721	0.836
PHL	0.944	0.942	0.911	0.965
POL	1.101	1.047	1.289	1.171
PRT	0.881	0.900	0.843	0.956
QAT	0.788	0.842	0.308	0.391
ROU	0.941	0.943	0.974	1.035
RUS	0.934	0.965	0.841	0.901
SGP	0.975	0.940	0.818	0.840
SVN	0.709	0.691	0.793	1.119
ESP	1.016	0.995	1.049	1.032
SWE	0.987	0.977	1.068	1.083
CHE	1.029	1.002	1.230	1.196
ARE	0.869	0.902	0.505	0.581
GBR	0.859	0.908	0.657	0.764
USA	1.081	1.019	1.119	1.035
ZAF	1.022	0.989	1.054	1.031
median	0.945	0.943	0.843	0.956
mean	0.937	0.923	0.878	0.919
std dev	0.122	0.108	0.251	0.204

This table present the welfare impact of a 20% decrease in bilateral financial frictions between China and other countries. Variables reported include cross-regime changes to countries' nominal wage ($\widehat{X}$, which equals output $\widehat{Y}$ based on Equation B.3), real wage ($\widehat{w}/\widehat{P}$), expenditure ($\widehat{X}$), and expenditure-to-output ratio ($\widehat{X}/\widehat{Y}$). A hat value higher (lower) than one means the variable's steady-state value in the counterfactual scenario under the policy change is greater (smaller) than in the original scenario observed in the data.

general equilibrium. Appendix B.3 describes the algorithm to solve the fixed point problem. Appendix B.4 discusses the existence and uniqueness of the solution. Appendix B.5 describes the calibration strategy for productivity shocks.

B.1 Exact Hat Algebra

This section applies the 'exact hat algebra' method developed by Dekle et al. (2007) (DEK) to characterize inter-regime changes of the model. A variable marked with a hat in this paper denotes the ratio of its counterfactual to original steady-state value:

$$\widehat{A} = \frac{\bar{A}^{ctf}}{\bar{A}^{org}}, \qquad \text{which suggests} \qquad \bar{A}^{ctf} = \bar{A}^{org}\widehat{A}. \tag{B.1}$$

On the production side, labor supply is inelastic and labor share in output is constant:

$$\bar{L}_i^{org} = \bar{L}_i^{ctf}, \qquad \frac{\bar{w}_i^s \bar{L}_i^s}{\bar{Y}_i^s} = (1-\mu)\eta, \;\; s = \{org, ctf\} \tag{B.2}$$

hence a country's inter-regime output change equals its wage change:

$$\widehat{Y}_i = \frac{\bar{Y}_i^{ctf}}{\bar{Y}_i^{org}} = \frac{\bar{w}_i^{ctf}\bar{L}_i^{ctf}}{\bar{w}_i^{org}\bar{L}_i^{org}} = \frac{\bar{w}_i^{org}\widehat{w}_i}{\bar{w}_i^{org}} = \widehat{w}_i. \tag{B.3}$$

Similarly for physical capital, its share in output is also constant

$$\frac{\bar{r}_i^s \bar{K}_i^s}{\bar{Y}_i^s} = \mu\eta, \tag{B.4}$$

which suggests that its rental fee satisfies

$$\widehat{Y}_i = \frac{\bar{Y}_i^{ctf}}{\bar{Y}_i^{org}} = \frac{\bar{r}_i^{ctf}\bar{K}_i^{ctf}}{\bar{r}_i^{org}\bar{K}_i^{org}} = \widehat{r}_i\widehat{K}_i. \tag{B.5}$$

Moreover, the Euler equation for capital (9) in the steady state is

$$\frac{\mu\eta\bar{Y}_i}{\bar{P}_i\bar{K}_i} = \frac{1}{\beta} + \delta - 1, \qquad \text{which suggests} \qquad \widehat{Y}_i = \frac{\bar{Y}_i^{ctf}}{\bar{Y}_i^{org}} = \frac{\bar{P}_i^{ctf}\bar{K}_i^{ctf}}{\bar{P}_i^{org}\bar{K}_i^{org}} = \widehat{P}_i\widehat{K}_i. \tag{B.6}$$

It follows from B.5 and B.6 that capital rental fee changes at the same rate as price

$$\widehat{r}_i = \widehat{P}_i. \tag{B.7}$$

The price in country i is given by Equations 3 and 4 whose steady state in regime s is

$$(\bar{P}_i^s)^{-\theta} = \Gamma^{-\theta}\sum_{j=1}^{I} T_j(\tau_{ji}^s)^{-\theta}[(\bar{r}_j^s)^{\mu\eta}(\bar{w}_j^s)^{(1-\mu)\eta}(\bar{P}_j^s)^{1-\eta}]^{-\theta}. \tag{B.8}$$

The counterfactual steady-state price ($\bar{P}_i^{ctf}$) from this equation, with the definition of a hat variable in B.1, can be written as

$$(\bar{P}_i^{org}\widehat{P}_i)^{-\theta} = \Gamma^{-\theta}\sum_{j=1}^{I} T_j(\tau_{ji}^{org}\widehat{\tau}_{ji})^{-\theta}[(\bar{r}_j^{org}\widehat{r}_j)^{\mu\eta}(\bar{w}_j^{org}\widehat{w}_j)^{(1-\mu)\eta}(\bar{P}_j^{org}\widehat{P}_j)^{1-\eta}]^{-\theta}. \tag{B.9}$$

Dividing the whole equation by $(\bar{P}_i^{org})^{-\theta}$, with the expression of the original steady-state price also obtained from B.8, yields the cross-regime price change

$$\widehat{P}_i^{-\theta} = \sum_{j=1}^{I}\bar{\pi}_{ji}^{org}\widehat{\tau}_{ji}^{-\theta}(\widehat{r}_j^{\mu\eta}\widehat{w}_j^{(1-\mu)\eta}\widehat{P}_j^{1-\eta})^{-\theta} = \sum_{j=1}^{I}\bar{\pi}_{ji}^{org}\widehat{\tau}_{ji}^{-\theta}(\widehat{w}_j^{(1-\mu)\eta}\widehat{P}_j^{\mu\eta+1-\eta})^{-\theta}. \tag{B.10}$$

Meanwhile, the wage change is derived from the goods market clearing condition in the counterfactual regime (Equation 5):

$$\bar{Y}_i^{ctf} = \sum_{j=1}^{I}\bar{\pi}_{ij}^{ctf}\bar{X}_j^{ctf}. \tag{B.11}$$

Expenditure $\bar{X}_i^{ctf}$ in this equation is linked to the country's wealth $\check{D}_i^{ctf}$ through the wealth constraint (Equation 17), which in the counterfactual steady state follows

$$\check{D}_i^{ctf}\bar{Y}_i^{ctf} = \check{D}_i^{ctf}\bar{Y}_i^{ctf}\frac{1}{\bar{\beta}} + \bar{Y}_i^{ctf} - \bar{X}_i^{ctf}, \tag{B.12}$$

where $\check{D}_i^{ctf}$ is normalized by a country's output as in Equation 22 and the steady-state discount factor $\bar{\beta}$ is the inverse of equilibrium asset return. It follows from B.12 that counterfactual expenditure is

$$\bar{X}_i^{ctf} = \bar{Y}_i^{ctf}[1 - \check{D}_i^{ctf}(1 - \frac{1}{\bar{\beta}})]. \tag{B.13}$$

In this expression, output changes at the same rate as wage (B.3) and therefore

$$\bar{Y}_i^{ctf} = \widehat{Y}_i\bar{Y}_i^{org} = \widehat{w}_i\bar{Y}_i^{org}. \tag{B.14}$$

Meanwhile, bilateral trade shares in the steady state based on Equations 3-4 are

$$\bar{\pi}_{ij}^{ctf} = \frac{\bar{T}_i[\tau_{ij}^{ctf}(\bar{r}_i^{ctf})^{\mu\eta}(\bar{w}_i^{ctf})^{(1-\mu)\eta}(\bar{P}_i^{ctf})^{1-\eta}]^{-\theta}}{(\bar{P}_j^{ctf})^{-\theta}/\Gamma^{-\theta}} = \frac{\bar{\pi}_{ij}^{org}\widehat{\tau}_{ij}^{-\theta}(\widehat{r}_i^{\mu\eta}\widehat{w}_i^{(1-\mu)\eta}\widehat{P}_i^{1-\eta})^{-\theta}}{\sum_{k=1}^{I}\bar{\pi}_{kj}^{org}\widehat{\tau}_{kj}^{-\theta}(\widehat{r}_k^{\mu\eta}\widehat{w}_k^{(1-\mu)\eta}\widehat{P}_k^{1-\eta})^{-\theta}} \tag{B.15}$$

Combining Equations B.7, B.13-B.15 in B.11 yields

$$\widehat{w}_i\bar{Y}_i^{org} = \sum_{j=1}^{I} \frac{\bar{\pi}_{ij}^{org}\widehat{\tau}_{ij}^{-\theta}(\widehat{w}_i^{(1-\mu)\eta}\widehat{P}_i^{\mu\eta+1-\eta})^{-\theta}}{\sum_{k=1}^{I}\bar{\pi}_{kj}^{org}\widehat{\tau}_{kj}^{-\theta}(\widehat{w}_k^{(1-\mu)\eta}\widehat{P}_k^{\mu\eta+1-\eta})^{-\theta}}\widehat{w}_j\bar{Y}_j^{org}[1-\check{D}_j^{ctf}(1-\frac{1}{\bar{\beta}})]. \tag{B.16}$$

B.10 and B.16 are the price determination and goods market clearing conditions (26 and 27) to derive all the countries' inter-regime changes of wages and prices. Together with intra-regime analysis that determines asset positions $\widehat{D}$, we can solve a joint fixed point problem of $(\widehat{w}, \widehat{P}, \widehat{D})$ to characterize the general equilibrium effects of policy changes.

B.2 Portfolio Choice Solution Method

This section applies the solution method developed by Devereux and Sutherland (2011) and Tille and van Wincoop (2010) (DSTW) to a multi-country portfolio choice problem with potential shifts of the steady state under policy changes. A variable marked with a tilde denotes the log-deviation of any variable A from its steady state value $\bar{A}$ under stochastic productivity shocks within a specific policy regime

$$\widetilde{A}_t = \ln(\frac{A_t - \bar{A}}{\bar{A}}). \tag{B.17}$$

Country i's portfolio is derived from the second-order Taylor expansion of its Euler equation (13)

$$E_t[\frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}}e^{-f_{i1}}R_{1,t+1}] = ... = E_t[\frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}}e^{-f_{iI\text{-}1}}R_{I\text{-}1,t+1}] = E_t[\frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}}e^{-f_{iI}}R_{I,t+1}] \tag{B.18}$$

while taking the difference between the numeraire asset I and all the other assets:

$$E_t[\widetilde{R}_{x,t+1} + \frac{1}{2}\widetilde{R}_{x,t+1}^2 - \widetilde{R}_{x,t+1}(\gamma\widetilde{C}_{i,t+1} + \widetilde{P}_{i,t+1})] = -\frac{1}{2}F_i + \mathcal{O}(\epsilon^3). \tag{B.19}$$

$R_{x,t+1}$ denotes a vector of excess returns relative to the numeraire asset

$$\widetilde{R}'_{x,t+1} = [\widetilde{R}_{1,t+1} - \widetilde{R}_{I,t+1}, \widetilde{R}_{2,t+1} - \widetilde{R}_{I,t+1}, ..., \widetilde{R}_{I\text{-}1,t+1} - \widetilde{R}_{I,t+1}], \tag{B.20}$$

$R_{x,t+1}^2$ denotes the vector of excess squared returns

$$\widetilde{R}_{x,t+1}^{2\prime} = [\widetilde{R}_{1,t+1}^2 - \widetilde{R}_{I,t+1}^2, \widetilde{R}_{2,t+1}^2 - \widetilde{R}_{I,t+1}^2, ..., \widetilde{R}_{I-1,t+1}^2 - \widetilde{R}_{I,t+1}^2], \tag{B.21}$$

and F_i denotes i's vector of financial frictions defined as

$$F'_i = [f_{iI} - f_{i1}, f_{iI} - f_{i2}, ..., f_{iI} - f_{iI\text{-}1}], \tag{B.22}$$

whose k^{th} element represents the additional financial friction country i's households incur when holding I's relative to k's asset. $\mathcal{O}(\epsilon^3)$ captures all terms of order higher than two.

The difference between country i's (B.19) and the numeraire country I's expanded Euler equation

$$E_t[\widetilde{R}_{x,t+1} + \frac{1}{2}\widetilde{R}^2_{x,t+1} - \widetilde{R}_{x,t+1}(\gamma\widetilde{C}_{I,t+1} + \widetilde{P}_{I,t+1})] = -\frac{1}{2}F_I + \mathcal{O}(\epsilon^3) \tag{B.23}$$

yields i's portfolio determination equation:

$$E_t[(\widetilde{C}^p_{i,t+1} - \widetilde{C}^p_{I,t+1})\widetilde{R}'_{x,t+1}] = \frac{1}{2}F_{iI} + \mathcal{O}(\epsilon^3), \qquad \forall i \in [1, I\text{-}1], \tag{B.24}$$

where $C^p_{i,t+1} = \frac{P_{i,t+1}}{C^{-\gamma}_{i,t+1}}$ is i's price- and utility-adjusted consumption which follows

$$\widetilde{C}^p_{i,t+1} = \gamma\widetilde{C}_{i,t+1} + \widetilde{P}_{i,t+1}, \tag{B.25}$$

and $\widetilde{C}^p_{i,t+1} - \widetilde{C}^p_{I,t+1}$ hence reflects the consumption differential between country i and the numeraire country I. F_{iI} denotes the excess financial frictions faced by i relative to by I

$$F_{iI} = F'_i - F'_I. \tag{B.26}$$

Equation B.24 is country i's portfolio determination equation: variables on its left are functions of i's asset positions, which are also affected by financial frictions on its right. $\breve{\alpha}_i$ denotes the vector of i's derived equilibrium bilateral asset positions adjusted for its output and discount factor

$$\breve{\alpha}_i = \frac{1}{\beta\bar{Y}_i}[\bar{\alpha}_{i1}, \bar{\alpha}_{i2}, ..., \bar{\alpha}_{iI\text{-}1}]. \tag{B.27}$$

Stacking Equation B.24 vertically, with each row representing a holder country, constructs a system of equations 19 for the world bilateral portfolio matrix 20 to be solved. If $\widetilde{C}^p_{x,t+1}$ denotes the vector of countries' adjusted consumption differentials relative to I's

$$\widetilde{C}^{p'}_{x,t+1} = [\widetilde{C}^p_{1,t+1} - \widetilde{C}^p_{I,t+1}, \widetilde{C}^p_{2,t+1} - \widetilde{C}^p_{I,t+1}, ..., \widetilde{C}^p_{I-1,t+1} - \widetilde{C}^p_{I,t+1}], \tag{B.28}$$

and F denotes the vector of countries' relative financial frictions

$$F^{'} = [F_{1I}, F_{2I}, ..., F_{I-1I}], \tag{B.29}$$

then the world portfolio matrix consisting of countries' (relative to I's) asset holdings

$$\breve{\alpha}^{'} = [\breve{\alpha}_1, \breve{\alpha}_2, ..., \breve{\alpha}_{I-1}] - \breve{\alpha}_I, \tag{B.30}$$

can be obtained from the system of portfolio determination equations in either policy regime s summarized by

$$E_t[\widetilde{C}^p_{x,t+1}\widetilde{R}'_{x,t+1}]^s = \frac{1}{2}F^s, \qquad s = \{org, ctf\}. \tag{B.31}$$

After solving non-numeraire countries' relative holdings, numeraire I's holdings can be obtained from the market clearing condition of each asset (Equation 15).

Consumption differential $\widetilde{C}^p_{x,t+1}$ is affected both directly and indirectly by stochastic productivity shocks of all the countries

$$\epsilon'_t = [\epsilon_{1,t}, \epsilon_{2,t}, ..., \epsilon_{I,t}]. \tag{B.32}$$

The direct impact happens since these shocks drive the fluctuations of macro fundamentals including consumption and price, and the indirect impact happens through these shocks' influence on excess financial returns $\breve{\alpha}\widetilde{R}_{x,t}$ which add to countries' wealth. Given this reasoning, we decompose $\widetilde{C}^p_{x,t+1}$ into two terms

$$\widetilde{C}^p_{x,t+1} = \widetilde{C}^d_{x,t+1} + \breve{\alpha}\widetilde{R}_{x,t+1}, \tag{B.33}$$

where $\widetilde{C}^d_{x,t+1}$ captures the part of $\widetilde{C}^p_{x,t+1}$ under the direct impact of productivity shocks, and $\breve{\alpha}\widetilde{R}_{x,t}$ captures the financial part under the indirect impact. Plugging B.33 in B.31 allows us to rewrite the portfolio determination equation as

$$E_t[(\widetilde{C}^d_{x,t+1} + \breve{\alpha}\widetilde{R}_{x,t+1})\widetilde{R}'_{x,t+1}]^s = E_t[\widetilde{C}^d_{x,t+1}\widetilde{R}'_{x,t+1} + \breve{\alpha}\widetilde{R}_{x,t+1}\widetilde{R}'_{x,t+1}]^s = \frac{1}{2}F^s. \tag{B.34}$$

To solve the portfolio choice problem, we evaluate the second moments $\widetilde{C}^d_{x,t+1}\widetilde{R}'_{x,t+1}$ and $\widetilde{R}_{x,t+1}\widetilde{R}'_{x,t+1}$ in Equation B.34. They represent the cross-country covariance of consumption differentials and asset returns, as well as the covariance matrix of asset returns respectively. To compute these two second moments, first we characterize the responses of $C^d_{x,t+1}$ and $R_{x,t+1}$ individually to productivity shocks:

$$\mathcal{D} = \frac{\partial C^d_{x,t+1}}{\partial \epsilon_{t+1}}, \qquad \mathcal{R} = \frac{\partial R_{x,t+1}}{\partial \epsilon_{t+1}}. \tag{B.35}$$

These are the coefficient matrices extracted from the first-order approximation of the model derived with linearization methods for DSGE models such as Blanchard and Kahn (1980), Uhlig (1995), and Sims (2002). Computing these coefficient matrices typically requires employing the linear method for the whole DSGE model to predict the first-order dynamics of control and state variables in response to the shocks in the economy. However, it is computationally challenging to implement when there are many countries with numerous variables and equations. Hu (2023a) describes a system reduction method for this macro-trade model. The main idea is to utilize the gravity trade model to characterize the intra-temporal allocation across many economies, so as to reduce the dynamic system and characterize the inter-temporal allocation efficiently.

After solving the DSGE model with linearization methods, we obtain the coefficient matrices in B.35. Their product together with the covariance matrix of productivity shocks Σ_T quantifies the second moments in Equation B.34, which follows

$$E_t[\widetilde{C}^d_{x,t+1}\widetilde{R}'_{x,t+1} + \check{\alpha}\widetilde{R}_{x,t+1}\widetilde{R}'_{x,t+1}]^s = E_t[\mathcal{D}\Sigma_T\mathcal{R}' + \check{\alpha}\mathcal{R}\Sigma_T\mathcal{R}']^s = \frac{1}{2}F^s. \tag{B.36}$$

We evaluate Equation B.36 in both original and counterfactual regimes:

$$E_t[(\mathcal{D}\Sigma_T\mathcal{R}')^{org} + \check{\alpha}^{org}(\mathcal{R}\Sigma_T\mathcal{R}')^{org}] = \frac{1}{2}F^{org}, \tag{B.37}$$

$$E_t[(\mathcal{D}\Sigma_T\mathcal{R}')^{ctf} + \check{\alpha}^{ctf}(\mathcal{R}\Sigma_T\mathcal{R}')^{ctf}] = \frac{1}{2}F^{ctf}. \tag{B.38}$$

The difference between B.37 and B.38 will predict changes to bilateral asset positions under the new second moments and financial frictions. If financial frictions remain unchanged in policy experiments, the friction matrices on the right hand side of these two equations cancel out, while portfolio changes influenced by second moments already consider the impacts of existing financial frictions reflected in $\check{\alpha}^{org}$. If financial frictions shift, we can derive portfolios affected by the difference of F matrix from these two equations. Lastly, we add up the changes in countries' bilateral holdings $\check{\alpha}$ to get the changes in their aggregate wealth positions $\check{D}$ across regimes.

Inter-regime changes of wealth $\widehat{D}$ will influence wage $\widehat{w}$ and price $\widehat{P}$ by shifting demand in the goods market. Meanwhile, asset positions $\widehat{D}$ are influenced by cross-country comovements of real variables $\widehat{w}, \widehat{P}$. They together with $\widehat{D}$ jointly determine the counterfactual steady state. Therefore, the solution to the joint fixed point problem of $(\widehat{w}, \widehat{P}, \widehat{D})$ captures the general equilibrium effects where financial and real sides of the world economy affect each other under policy changes.

B.3 Algorithm

Step 1. Calibrate the original steady state of the economy.

Obtain timed-averaged country-level output and wealth, as well as bilateral trade shares and bilateral portfolio weights from the data to calibrate the steady state of the original regime with $\bar{Y}^{org}, \check{D}^{org}, \bar{\pi}^{org}, \check{\alpha}^{org}$.

Step 2. Examine original portfolios and predict counterfactual frictions.

Loglinearize the model and apply DSTW's method around the original steady state to evaluate the portfolio Equation 28 and quantify the matrix of financial frictions F^{org}.[19]

[19]DSTW perform a first-order approximation of equations from the model to predict the responses of $\widetilde{C}^p_{x,t+1}$ and $\widetilde{R}_{x,t+1}$ to productivity shocks ϵ_{t+1}. Their product with the productivity covariance matrix

If there are any changes to bilateral trade costs or financial frictions in counterfactual exercises, obtain their matrices $\widehat{\tau}_{ij} = \tau_{ij}^{ctf}/\tau_{ij}^{org}$, $\widehat{F}_{ij} = F_{ij}^{ctf}/F_{ij}^{org}$.

Step 3. Form guesses about inter-regime changes under counterfactual frictions

$$(\widehat{w}^0, \widehat{P}^0, \widehat{D}^0). \tag{B.39}$$

They represent the vectors of all the countries' price, wage, and wealth changes under counterfactual policies. Use these changes together with original steady-state variables from Step 1 to predict the counterfactual steady state with $\bar{Y}^0, \check{D}^0, \bar{\pi}^0$.

Step 4. Solve for financial allocation in the counterfactual regime.

Apply DSTW's linear method around the counterfactual steady state from Step 3 and with financial frictions F^{ctf} from Step 2 to solve for countries' holdings of non-numeraire assets, including both numeaire and non-numeraire countries' holdings using 15, 28, 29:

$$\check{\alpha}_{ik}^1, \qquad i \in \{1, 2, ..., I\}, \qquad k \in \{1, 2, ..., I\text{-}1\}, \tag{B.40}$$

which will add to i's updated wealth based on 22:

$$\check{D}_i^1 = \bar{\beta} \sum_{k=1}^{I-1} \check{\alpha}_{ik}^1 + \bar{\beta} \check{\alpha}_{iI}^1. \tag{B.41}$$

This wealth determination equation also requires i's holding of the numeraire asset ($\check{\alpha}_{iI}$) undetermined by portfolio equations 28-29. The updating rule for $\check{\alpha}_{iI}$ is assumed to follow

$$\check{\alpha}_{iI}^1 = \zeta_D^1 \check{\alpha}_{iI}^0 + (1 - \zeta_D^1) \check{\alpha}_{iI}^*, \tag{B.42}$$

where $\zeta_D^1 \in [0, 1]$ is a weight using which $\check{\alpha}_{iI}^1$ is computed as a weighted sum of its value from the previous iteration $\check{\alpha}_{iI}^0$ as part of its initial wealth $\check{D}_i^0$,[20] and the difference between $\check{D}_i^0$ and i's updated holdings of non-numeraire assets in the current iteration $\check{\alpha}_{ik}^1$ from B.40:

$$\check{\alpha}_{iI}^0 = (\check{D}_i^0 - \bar{\beta} \sum_{k=1}^{I-1} \check{\alpha}_{ik}^0)/\bar{\beta}, \qquad \check{\alpha}_{iI}^* = (\check{D}_i^0 - \bar{\beta} \sum_{k=1}^{I-1} \check{\alpha}_{ik}^1)/\bar{\beta}. \tag{B.43}$$

To provide intuition for the updating rule B.42, let us consider two extreme cases where $\zeta_D^1 = 0$ or 1. If $\zeta_D^1 = 0$ which implies $\check{D}_i^1 = \check{D}_i^0$, households only adjust intra- but not inter-temporal financial allocations, because i's equilibrium wealth $\check{D}_i$ as its total asset position, decided by households' inter-temporal consumption-saving decisions, is

Σ_T, which appears as the second-moment variable $\widetilde{C}_{x,t+1}^p \widetilde{R}_{x,t+1}'$ on the left hand side of the portfolio equation, equals the matrix of financial frictions F^{org} on the right hand side of Equation 28. Appendix B.2 provides more details of the method.

[20] $\check{\alpha}_{iI}^0$ is set as the original value $\check{\alpha}^{org}$ in the first iteration. In the $m+1^{th}$ iteration, $\check{\alpha}_{iI}^m$ is solved from the m^{th} iteration.

unaffected by cross-regime changes of second moments in portfolio equations (28-29). If $\zeta_D^1 = 1$, portfolio changes shaped by these second moments completely translate to equilibrium wealth changes:

$$\check{\alpha}_{iI}^1 = \check{\alpha}_{iI}^0, \qquad \check{D}_i^1 - \check{D}_i^0 = \bar{\beta}(\sum_{k=1}^{I-1} \check{\alpha}_{ik}^1 - \sum_{k=1}^{I-1} \check{\alpha}_{ik}^0). \tag{B.44}$$

Households do not adjust $\check{\alpha}_{iI}$ intra-temporally to offset changes in non-numeraire asset holdings in this case, where second moments are fully reflected in inter-temporal adjustments of equilibrium wealth $\check{D}_i$.[21] Between these two extreme cases with $\zeta_D^1 \in (0, 1)$, the updating rule B.42 incorporates both inter- and intra-temporal financial adjustments to second moments across regimes.

In each iteration m, the value of ζ_D^m is determined by the market clearing condition of the numeraire asset I.[22] For example in the first iteration, ζ_D^1 is derived from B.42 to satisfy

$$\sum_{i=1}^{I} \check{\alpha}_{iI}^1 \bar{Y}_i^0 = 0. \tag{B.45}$$

After obtaining ζ_D^1 from B.45 and $\check{\alpha}_{iI}^1$ from B.42, follow B.41 to update inter-regime changes of equilibrium wealth $\widehat{D}^1$.

Step 5. Update changes to real variables given the solved financial allocation.

Update inter-regime changes of $\widehat{w}, \widehat{P}$ with $\widehat{D}^1$ from Step 4. This involves employing an iterative computation procedure by DEK who use a contraction mapping function M from $\widehat{w}^0$ to $\widehat{w}^1$ with a constant $\zeta_w \in (0, 1)$

$$\widehat{w}^1 = M(\widehat{w}^0) = \widehat{w}^0 (1 + \zeta_w \frac{\mathbb{Z}_i(\widehat{w}^0)}{\bar{Y}_i^{org}}), \tag{B.46}$$

where $\mathbb{Z}_i(\widehat{w})$ is excess demand derived from the goods market clearing condition 27:

$$\mathbb{Z}_i(\widehat{w}_i) = \frac{1}{\widehat{w}}[\widehat{w}_i \bar{Y}_i^{org} - \sum_{j=1}^{I} \frac{\bar{\pi}_{ij}^{org} \widehat{\tau}_{ij}^{-\theta} (\widehat{w}_i^{(1-\mu)\eta} \widehat{P}_i^{\mu\eta+1-\eta})^{-\theta}}{\sum_{k=1}^{I} \bar{\pi}_{kj}^{org} \widehat{\tau}_{kj}^{-\theta} (\widehat{w}_k^{(1-\mu)\eta} \widehat{P}_k^{\mu\eta+1-\eta})^{-\theta}} \widehat{w}_j \bar{Y}_j^{org} (1 - \check{D}_j^{org} \widehat{D}_j^1 (1 - \frac{1}{\bar{\beta}}))], \tag{B.47}$$

and solve for the corresponding $\widehat{P}^1$ to $\widehat{w}^1$ with the price determination equation (26). The mapping function M is bounded by one under a normalization condition that treats

[21] In this case, wealth adjustment can be very volatile from iteration to iteration, which is also self-fulfilling because wealth shifts the demand system in the goods market to drastically move cross-country covariances reflected as second moments in portfolio equations.

[22] This market clearing condition of asset I, together with those of other assets $k \in \{1, 2, ..., I\text{-}1\}$, holds in all the iterations. Using these conditions, we characterize financial reallocation across countries driven by their demand for assets influenced by the changes in second moments across policy regimes subject to the world resource constraint.

the world output as a numeraire and the world resource constraint[23]

$$\sum_{i=1}^{I} \widehat{w} \bar{Y}_i^{org} = 1, \qquad \sum_{i=1}^{I} \bar{Y}_i^{ctf} = \sum_{i=1}^{I} \bar{X}_i^{ctf}, \tag{B.48}$$

which enables the procedure to converge to the model solution.

Step 6. Repeat Steps 3-5 until convergence.

Use $\widehat{D}^1$ from Step 4 and $\widehat{w}^1$, $\widehat{P}^1$ from Step 5 as new guesses, repeat Steps 3-5 with both inter- and intra-regime analyses to reach new $\widehat{w}^2$, $\widehat{P}^2$, $\widehat{D}^2$. This continues until the wage difference between the m^{th} and the $m{+}1^{th}$ iteration $\mid \widehat{w}^{m+1}$-$\widehat{w}^m \mid$ is sufficiently small,[24] which solves the joint fixed point problem of ($\widehat{w}$, $\widehat{P}$, $\widehat{D}$) to characterize the counterfactual steady state under alternative trade costs or financial frictions.

B.4 Properties of the Solution

The existence and uniqueness of a model solution characterized by the 'exact hat algebra' technique have been well established in the trade literature. For example, DEK characterize the solution to wage changes $\widehat{w}$ given counterfactual asset positions $\check{D}^{ctf}$ under global trade balance. They follow the theorems by Alvarez and Lucas (2007), who show that under the assumptions that $\eta < 1$, $1 + \theta(1-\epsilon) > 0$, $\tau \geq 1$, a unique solution to $\widehat{w}$ exists to ensure zero excess demand in the goods market:

$$\mathbb{Z}_i(\widehat{w}_i) = \frac{1}{\widehat{w}_i}[\widehat{w}_i \bar{Y}_i^{org} - \sum_{j=1}^{I} \frac{\bar{\pi}_{ij}^{org} \widehat{\tau}_{ij}^{-\theta} (\widehat{w}_i^{(1-\mu)\eta} \widehat{P}_i^{\mu\eta+1-\eta})^{-\theta}}{\sum_{k=1}^{I} \bar{\pi}_{kj}^{org} \widehat{\tau}_{kj}^{-\theta} (\widehat{w}_k^{(1-\mu)\eta} \widehat{P}_k^{\mu\eta+1-\eta})^{-\theta}} \widehat{w}_j \bar{Y}_j^{org} (1 - \check{D}_j^{ctf} (1 - \frac{1}{\bar{\bar{\beta}}}))]. \tag{B.49}$$

Their required conditions to derive unique counterfactual wage include $\mathbb{Z}_i(w)$ is continuous, homogenous of degree zero, has the gross substitute property $\frac{\partial \mathbb{Z}_i(w)}{\partial w_j} > 0$, satisfies Walras's Law ($\sum_i w_i \mathbb{Z}_i(w) = 0$), faces a lower but not upper bound $\mathbb{Z}_i(w) >$ - $\max_j L_j, \max_i \mathbb{Z}_i(w \rightarrow w^{org}) \rightarrow \infty$. Most of these properties are maintained under the assumptions specified in this model. For example, if all the asset market clearing conditions (15) hold, the world resource constraint is satisfied:

$$\sum_{i=1}^{I} \bar{Y}_i^{ctf} = \sum_{i=1}^{I} \bar{X}_i^{ctf} = \sum_{i=1}^{I} \bar{Y}_i^{ctf} [1 - \check{D}_j^{ctf} (1 - \frac{1}{\bar{\bar{\beta}}})]. \tag{B.50}$$

[23] The world resource constraint automatically holds as long as all the asset clearing conditions are satisfied when solving for financial allocation (B.40 and B.45).

[24] See a discussion on the existence and uniqueness of the solution to the model in Appendix B.4.

As a result, Walras's Law is satisfied:

$$\sum_{i=1}^{I} \widehat{w}_i \mathbb{Z}_i(\widehat{w}) = \sum_{i=1}^{I} (\widehat{w}_i \bar{Y}_i^{org} - \sum_{j=1}^{I} \bar{\pi}_{ij}^{ctf} \widehat{w}_j \bar{Y}_j^{org} [1 - \check{D}_j^{ctf} (1 - \frac{1}{\bar{\beta}})]) = 0, \tag{B.51}$$

which is necessary to establish the existence of the solution to inter-regime changes.

The main departure of this model from DEK lies in the fact that counterfactual asset positions $\check{D}_j^{ctf}$ are exogenous and fixed in their analysis, but endogenous and variable in this model. This wealth position is characterized by Step 4 in the algorithm section after obtaining ζ_D from B.45 and $\check{\alpha}_{iI}$ from B.42. In the two policy experiments conducted in this paper, ζ_D always starts with positive numbers and eventually approaches zero as iterations proceed. $\zeta_D = 0$ represents the case where wealth is fixed as it is not updated across iterations. In that case, real variables are updated in the same way as in DEK to converge to the unique solution of the model.

B.5 Calibration of Productivity

I follow Levchenko and Zhang (2014), who infer Ricardian productivity from bilateral trade data, when estimating country-level productivity consistent with the Eaton and Kortum (2002) (EK) model.

Let country i's production cost be denoted as

$$c_{i,t} = (r_{i,t}^{\mu} w_{i,t}^{1-\mu})^{\eta} P_{i,t}^{1-\eta}. \tag{B.52}$$

It follows from Equation 3 that trade shares for any destination country j should satisfy

$$\frac{\pi_{ij,t}}{\pi_{jj,t}} = \frac{T_{i,t}}{T_{j,t}} (\frac{\tau_{ij,t} c_{i,t}}{c_{j,t}})^{-\theta}. \tag{B.53}$$

As the left hand side is directly observable from the trade data, I can recover relative productivity $\frac{T_{i,t}}{T_{j,t}}$ after estimating bilateral trade friction $\tau_{ij,t}$ and relative input cost $\frac{c_{i,t}}{c_{j,t}}$.

I follow the trade literature by estimating bilateral trade costs $\tau_{ij,t}$ from a combination of gravity variables sourced from CEPII including geographic distance divided into intervals set by EK, dummies for contiguity, common language, colonizer, religion, legal system, and regional trade agreements.

I estimate a country's production cost ($c_{i,t}$) based on the information from the PWT. Specifically, I compute a country's wage (w) as the ratio of its total labor compensation (output-side GDP ($rgdpo$) $\times$ share of labor compensation in GDP ($labsh$)) to total labor hours (number of employees (emp) $\times$ average hours per employee (avc)). Price of domestic absorption (pl_{da}) and price of capital services (pl_k) are used as the proxies for the price of intermediate inputs and capital rental fee respectively. Besides, I calibrate the share of intermediate input in production $\eta = .312$ as DEK and the share of labor input $1-\mu$ as country-specific $labsh$ from the PWT. The production cost of ROW is calculated as the median cost across countries not included in Table A.1.

The full estimating specification for all the country pairs in the sample follows

$$\ln(\frac{\pi_{ij,t}}{\pi_{jj,t}}) = \ln(T_{i,t}c_{i,t}^{-\theta}) - \ln(T_{j,t}c_{j,t}^{-\theta}) - \theta\tau_{ij,t} + \gamma_{ij,t}, \tag{B.54}$$

The first two terms on the right $\ln(T_{i,t}c_{i,t}^{-\theta})$ and $\ln(T_{j,t}c_{j,t}^{-\theta})$ can be captured by the exporter and importer fixed effects respectively when running the estimation. $\tau_{ij,t}$ represents the estimated bilateral trade costs as a linear combination of the gravity variables described above and $\gamma_{ij,t}$ stands for error terms. Exponentiating the importer fixed effects yields a term that combines country j's productivity and cost denoted as

$$Tc_{j,t} = T_{j,t}c_{j,t}^{-\theta}. \tag{B.55}$$

If the US is the benchmark country whose productivity ($T_{US,t}$) is its TFP value from the PWT ($rtfpna$). Then other countries' Ricardian productivity can be calculated as

$$T_{j,t} = T_{US,t}\frac{Tc_{j,t}}{Tc_{US,t}}(\frac{c_{j,t}}{c_{US,t}})^{\theta}, \tag{B.56}$$

where trade elasticity $\theta = 4$ following Simonovska and Waugh (2014). After calculating countries' dynamic productivity $T_{j,t}$, I estimate the persistence parameter in the AR(1) process to be 0.85 from all the countries in the sample and obtain country-specific time-averaged productivity $\bar{T}_j$. I then use these persistence and mean values in Equation 2 to recover productivity innovations and their cross-country covariance matrix Σ_T.